2 Open short

2.1 Trader deposit

For some number of bonds being shorted, Δy, the short deposit in shares is made up of several components:

- The **total value** in shares that underlies the bonds: $V(\Delta y) = (\frac{c_1}{c_0} + \phi_f) \cdot \frac{\Delta y}{c}$
- The **curve fee**: $\Phi_{\text{c,os}}(\Delta y) = \phi_c \cdot (1 - p) \cdot \frac{\Delta y}{c}$
- The **short principal**: $S(\Delta y) = z - \frac{1}{\mu} \cdot (\frac{\mu}{c} \cdot (k - (y + \Delta y)^{1-t_s}))^{\frac{1}{1-t_s}}$

where p is the **spot price**, which defines the relationship between shares and bonds when making an infinitesimally small trade. It is given by:

$$p = \left(\frac{\mu z_e}{y}\right)^{t_s} \tag{2}$$

The short principal is computed via the yieldspace function `yieldspace.calculate_shares_out_given_bonds_in_down(delta_y)`.
Finally, we need to define the **short proceeds**, which are the "proceeds in shares of closing a short position":

$$\text{short\_proceeds} = \left(\frac{c_1}{c_0} + \phi_f\right) \cdot \frac{\Delta y}{c} - \Delta z \tag{3}$$

The trader opening a short has to pay a **deposit**, which is their maximum loss when closing the short. This requires us to use the short proceeds and define $\Delta z = S(\Delta y) - \Phi_{\text{c,os}}(\Delta y)$, giving us
`deposit = total_value - short_principal + curve_fee` when in a valid regime, or

$$D(\Delta y) = \begin{cases} V(\Delta y) - S(\Delta y) + \Phi_{c,os}(\Delta y), & \text{if } V(\Delta y) > S(\Delta y) - \Phi_{c,os}(\Delta y) \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Where the cases avoid situations with negative interest (and thus place us in a valid regime). For some deposit, the realized price for the trader is then

$$p_r = 1 - \frac{D(\Delta y)}{\Delta y} \tag{5}$$

LPs take the opposite side of trades, so when a trader opens a short the LP opens a long. The short principal is the price paid by the LP to buy bonds, which are then set aside as if "sold" by the trader when they opened their short. The price the LP paid is

$$p_s = \frac{S(\Delta y)}{\Delta y} \tag{6}$$

2.2 Defining a conservative short deposit

If we are given a deposit amount, D, then calculating the bonds shorted is difficult due to the non-linear LP short principal component, $S(\Delta y)$. We have to approximate this in order to define a lower-bound on the short deposit. This is achieved by considering that Hyperdrive has a minimum allowable trade size, `min_tx`. Coupled with an upper bound of the number of available shares in the pool reserves, we have:

$$S(\text{min\_tx}) \leq S(\Delta y) \leq z_e - z_{\min} \tag{7}$$

Next, we define a constant:

$$A = \frac{c_1}{c_0} + \phi_f + \phi_c \cdot (1 - p) \tag{8}$$

This gives us a more compressed version of the deposit equation to make clear a conservative estimate using the bounded short principal:

$$\begin{aligned}
D(\Delta y) &= A \cdot \frac{\Delta y}{c} - S(\Delta y) \\
S(\Delta y) &\geq S(\text{min_tx}) \\
&\therefore \\
D(\Delta y) &\leq A \cdot \frac{\Delta y}{c} - S(\text{min_tx}) \\
\Delta y(D) &\geq \frac{c}{A} \cdot (D + S(\text{min_tx}))
\end{aligned} \tag{9}$$