We use real numbers in this document, as well as the math division (not //). Let's write

$$Q = \frac{totalAssets + VASSETS}{totalShares + VSHARES} << 1$$

.

1 Withdraw

1.1 Inequalities

we have:

$$assets = \lfloor shares \times Q \rfloor$$

so:

$$asset \leq shares * Q$$

$$\frac{asset}{Q} \leq shares$$

but also:

$$shares \times Q < asset + 1$$

$$shares < \frac{asset + 1}{Q}$$

so:

$$(shareMin = \left\lfloor \frac{asset}{Q} \right\rfloor) \leq shares < (shareMax = \left\lceil \frac{asset + 1}{Q} \right\rceil)$$

1.2 Lemmas

Lemma 1.

$$assets - 1 \leq \lfloor sharesMin \times Q \rfloor$$

Proof.

$$\begin{aligned} A &= \lfloor sharesMin \times Q \rfloor \\ &= \left\lfloor \left\lfloor \frac{assets}{Q} \right\rfloor \times Q \right\rfloor \\ &\geq \left\lfloor \left(\frac{assets}{Q} - 1 \right) \times Q \right\rfloor \\ &\geq \lfloor assets - Q \rfloor \\ &\geq assets - 1 \end{aligned}$$

Because $Q << 1$ and $assets \in \mathbf{N}$ □

Lemma 2.

$$assets - 1 \leq \lfloor sharesMax \times Q \rfloor$$

Proof.

$$
\begin{aligned}
A &= \lfloor sharesMax \times Q \rfloor \\
&= \left\lfloor \left\lceil \frac{asset+1}{Q} \right\rceil \times Q \right\rfloor \\
&\leq \left\lfloor \frac{asset+1}{Q} \times Q \right\rfloor \\
&\leq \lfloor asset+1 \rfloor \\
&\leq asset+1
\end{aligned}
$$

Because $assets \in \mathbf{N}$ ∎

Lemma 3.

$$
\left\lfloor \left\lfloor \frac{sharesMin + sharesMax}{2} \right\rfloor \times Q \right\rfloor = assets
$$

Proof.

$$
A = \left\lfloor \left\lfloor \frac{sharesMin + sharesMax}{2} \right\rfloor \times Q \right\rfloor
$$

$$
A = \left\lfloor \left\lfloor \frac{\left\lfloor \frac{asset}{Q} \right\rfloor + \left\lceil \frac{asset+1}{Q} \right\rceil}{2} \right\rfloor \times Q \right\rfloor
$$

So, on the one hand:

$$
\begin{aligned}
A &\geq \left\lfloor \frac{\left\lfloor \frac{asset}{Q} \right\rfloor + \left\lceil \frac{asset+1}{Q} \right\rceil}{2} \times Q \right\rfloor \\
A &\geq \left\lfloor \frac{\frac{asset}{Q} - 1 + \frac{asset+1}{Q}}{2} \times Q \right\rfloor \\
A &\geq \left\lfloor asset + \frac{1-Q}{2} \right\rfloor \\
A &\geq asset
\end{aligned}
$$

And, on the other hand:

$$
\begin{aligned}
A &\leq \left\lfloor \left(\frac{\left\lfloor \frac{asset}{Q} \right\rfloor + \left\lceil \frac{asset+1}{Q} \right\rceil}{2} - 1 \right) \times Q \right\rfloor \\
A &\leq \left\lfloor \left(\frac{\frac{asset}{Q} + \frac{asset+1}{Q} + 1}{2} - 1 \right) \times Q \right\rfloor \\
A &\leq \left\lfloor asset + \frac{1-Q}{2} \right\rfloor \\
A &\leq asset
\end{aligned}
$$

So $A = asset$. $\square$