Glaciology
EESCGU4220

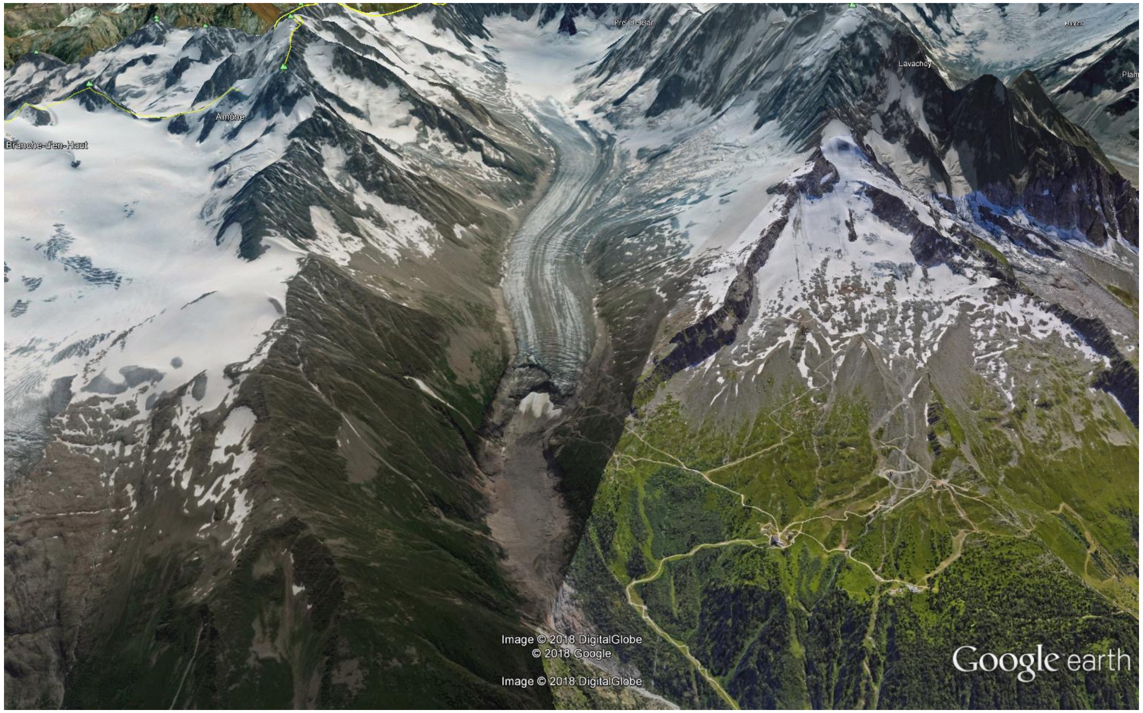
Lecture 7: Basal slip

Glaciers slide:
How do we know?

What controls how fast this happens?

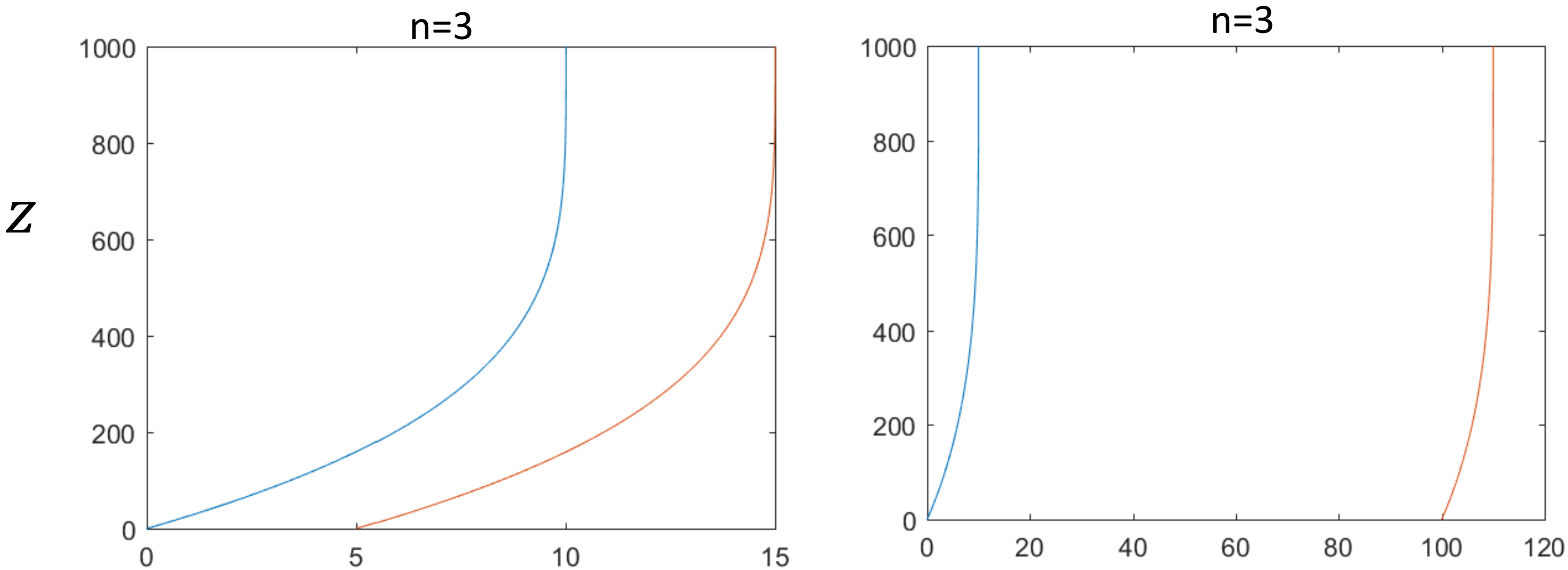
How do we model slip?

Implications?



https://www.youtube.com/watch?v=njTjfJcAsBg

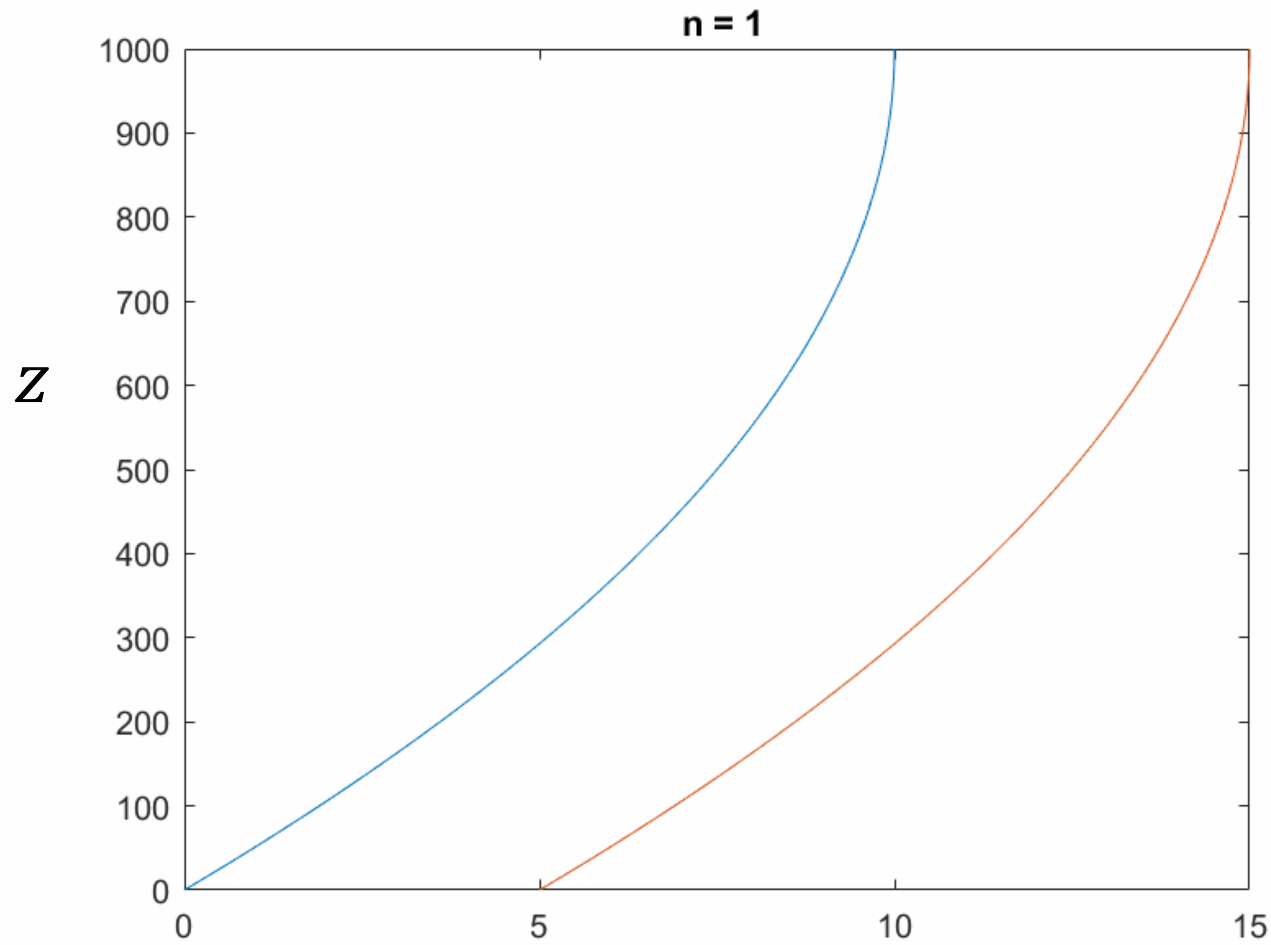
"Sliding" and "basal slip"



$$u(z) = u_{sia}\left(1 - \left(1 - \frac{z}{H}\right)^{n+1}\right)$$

$$u(z) = u_b + u_{sia}\left(1 - \left(1 - \frac{z}{H}\right)^{n+1}\right)$$

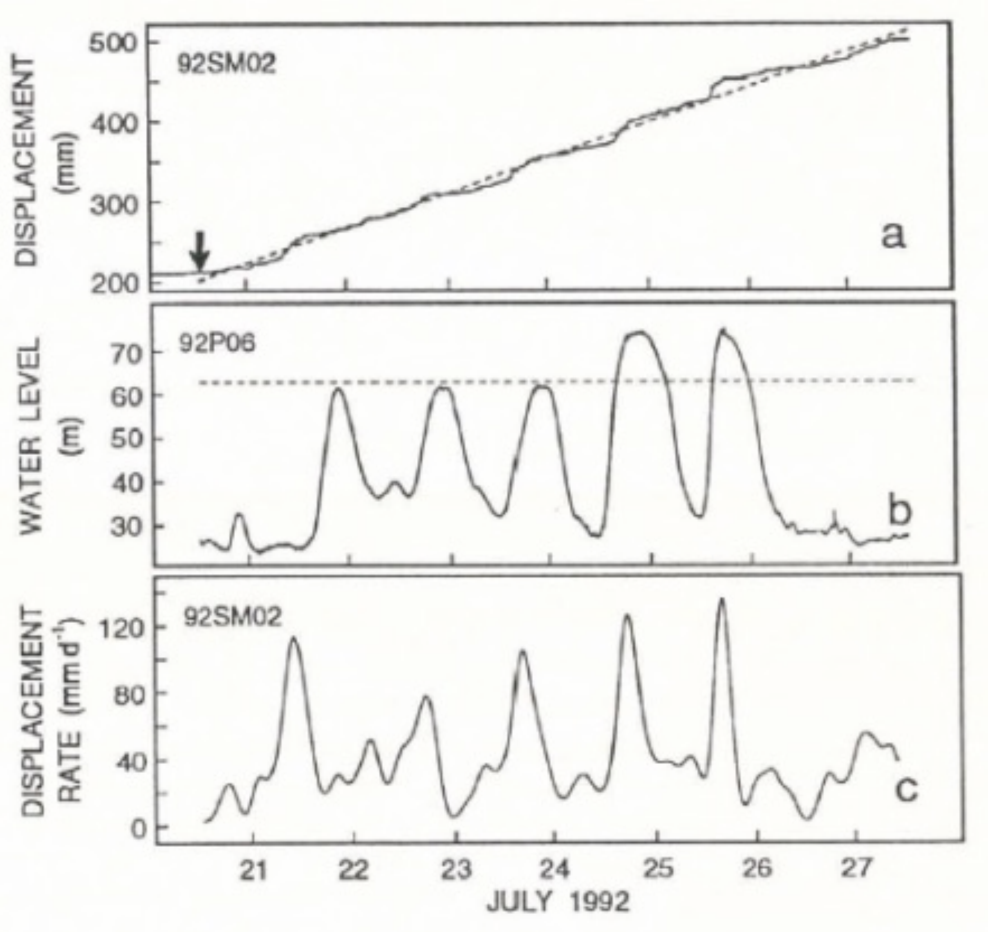
“Sliding” and “basal slip”

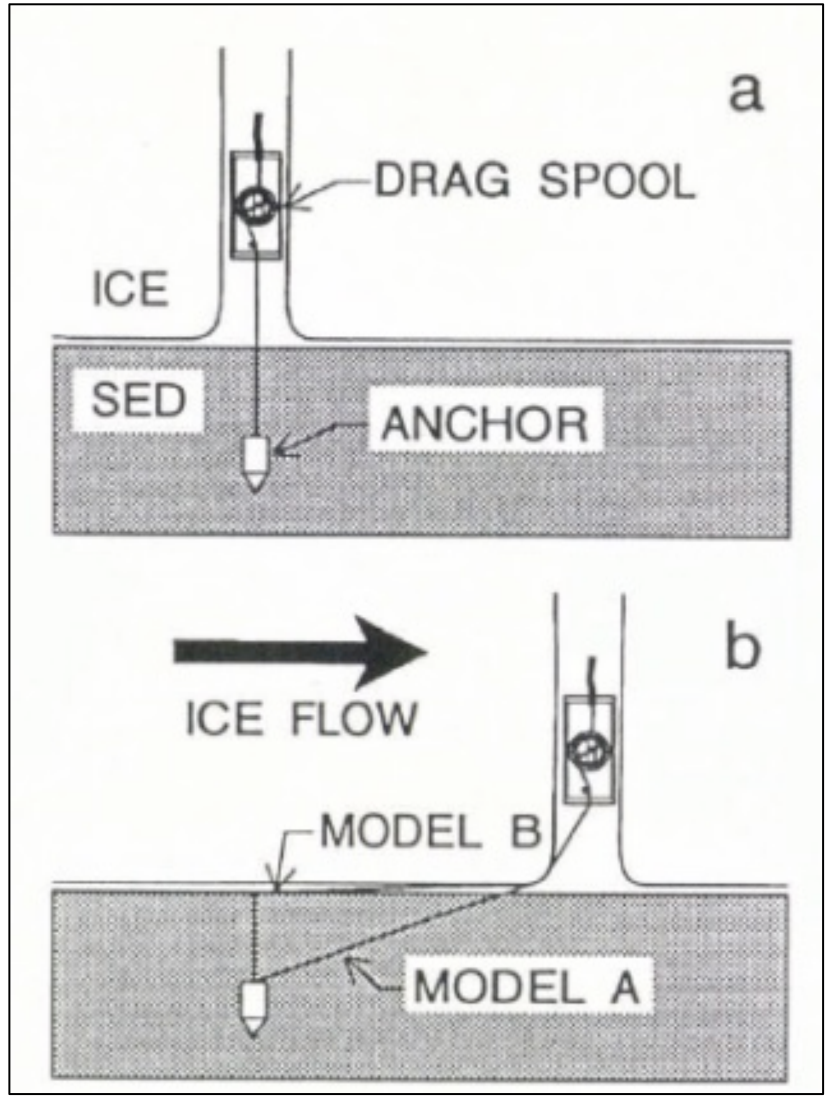


$$u(z) = u_b + u_{sia}\left(1 - \left(1 - \frac{z}{H}\right)^{n+1}\right)$$

How do we know?

1. Direct observations





50-70% was slip, the rest was sediment deformation.

Blake et al. (1994)

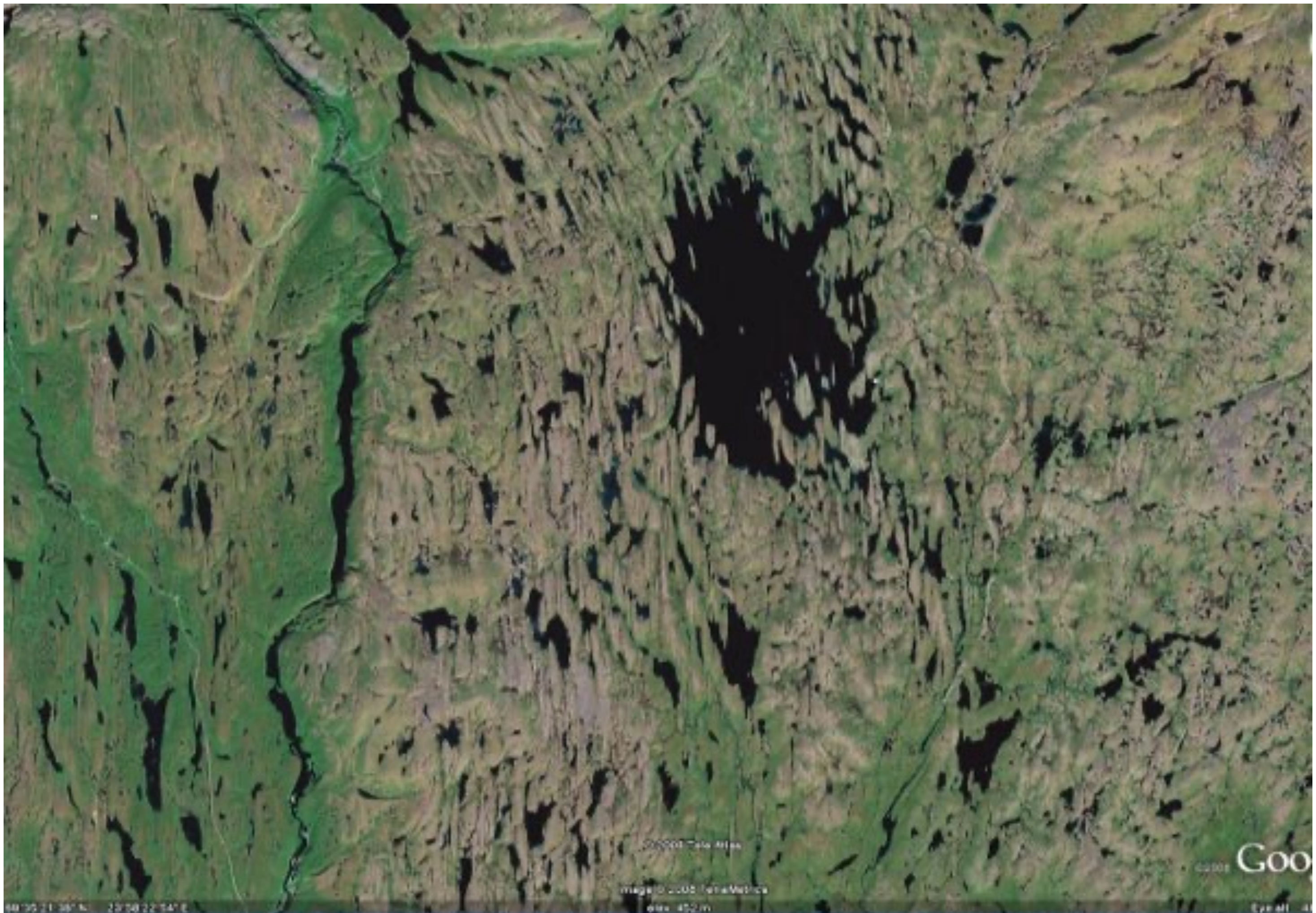
How do we know?

2. Bedforms

Chris Clark's website: https://www.sheffield.ac.uk/drumlins/drumlins

How do we know?

2. Bedforms



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How do we know?

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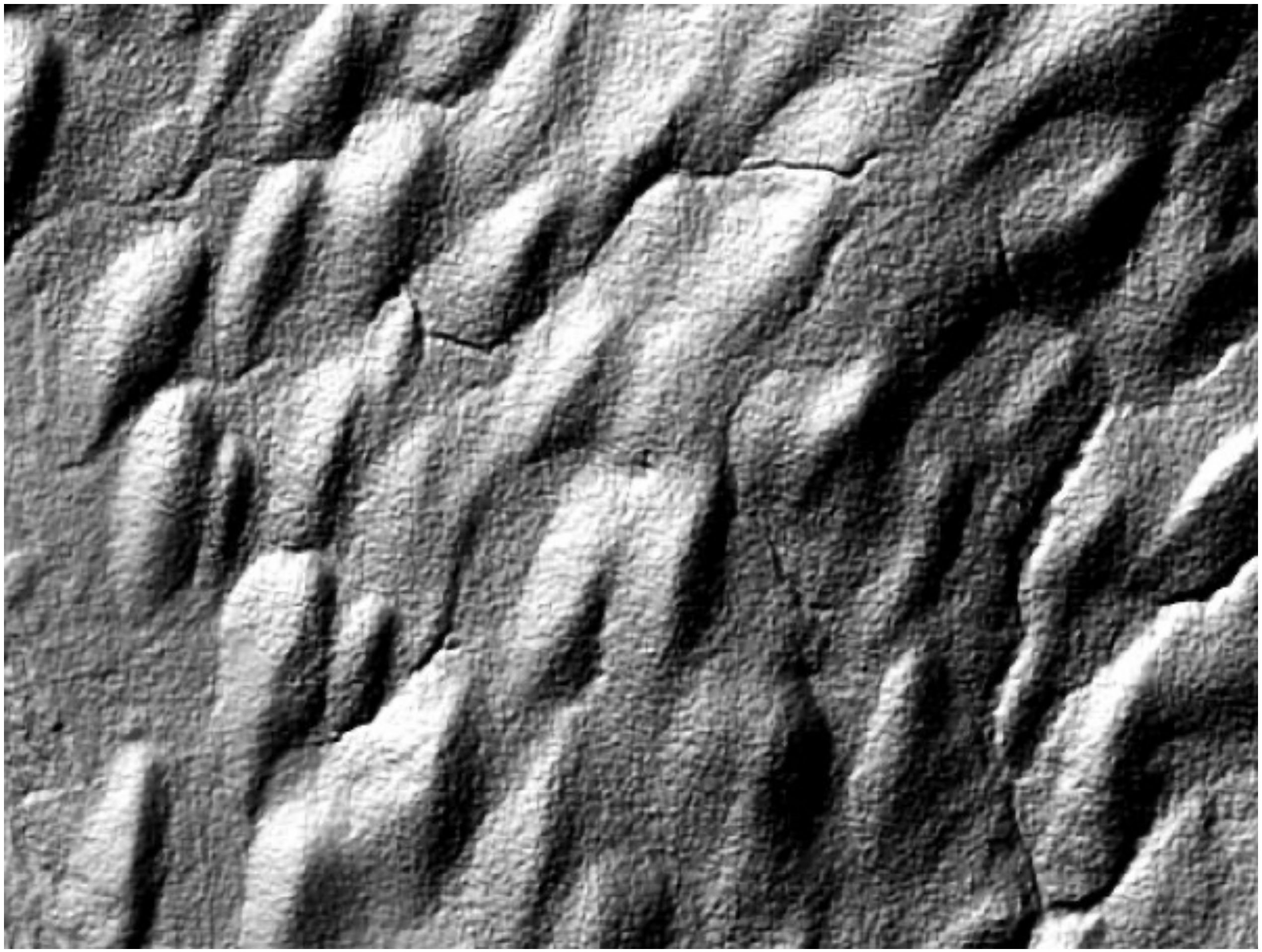

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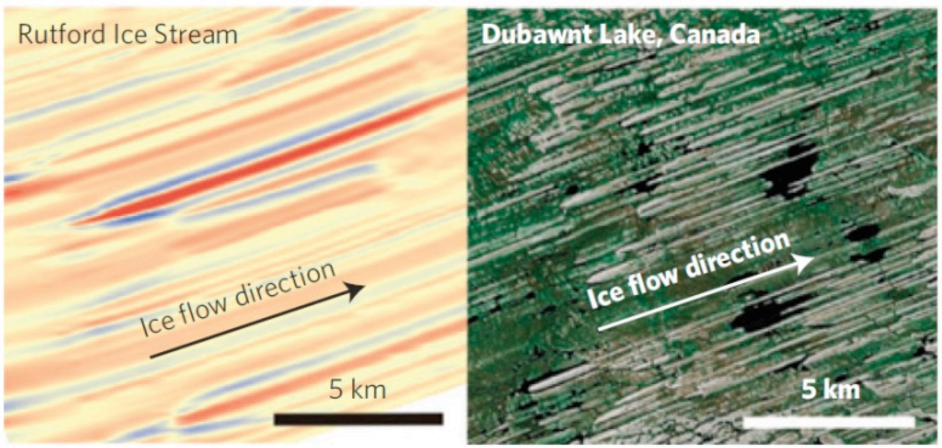


King et al. (2009)

Chris Clark's website: https://www.sheffield.ac.uk/drumlins/drumlins

How do we know?

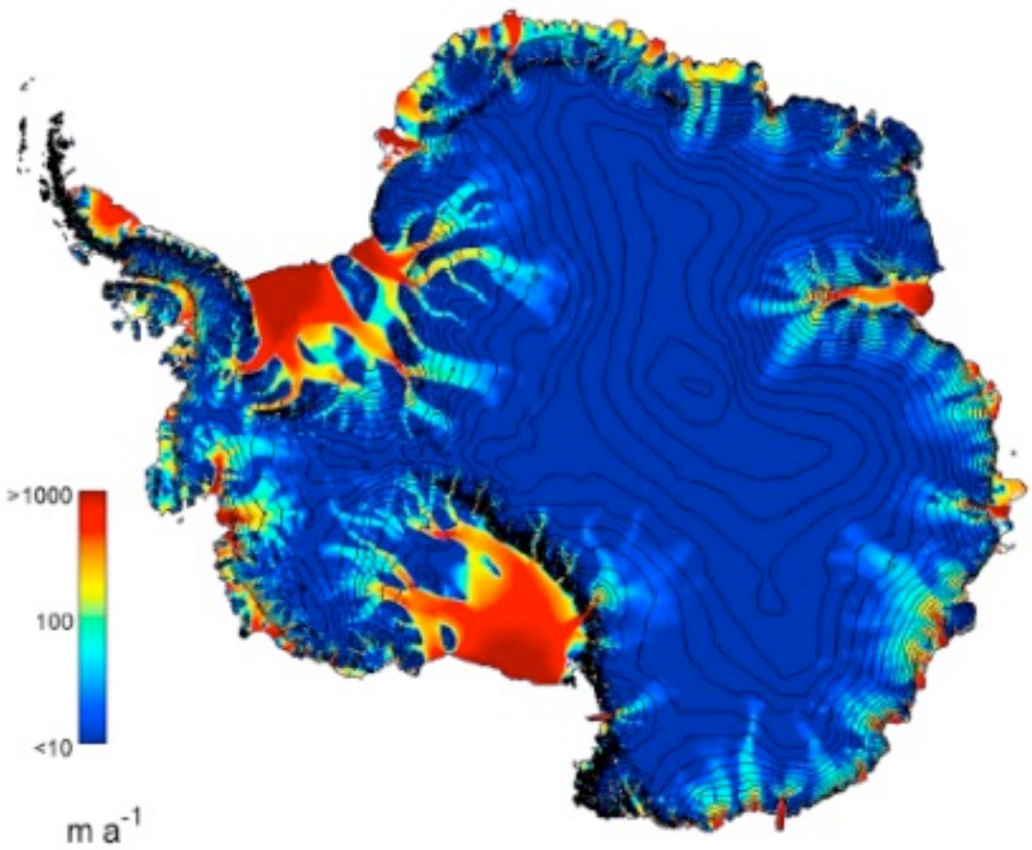
2. Bedforms

Chris Clark's website: https://www.sheffield.ac.uk/drumlins/drumlins

How do we know?

3. Inversions

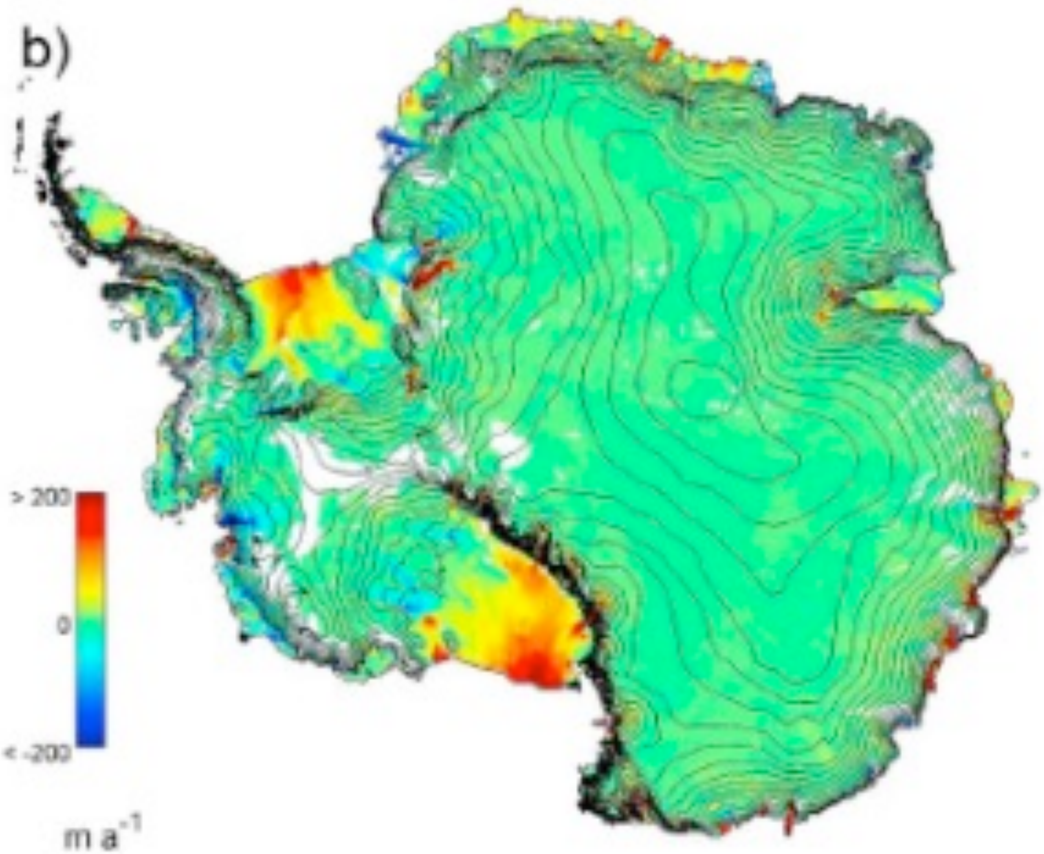
Modelled flow speed [m/yr]



Arthern et al. (2015)

How do we know?

3. Inversions



Difference between obs. and modelled flow speed [m/yr]

Arthern et al. (2015)

How do we know?

3. Inversions

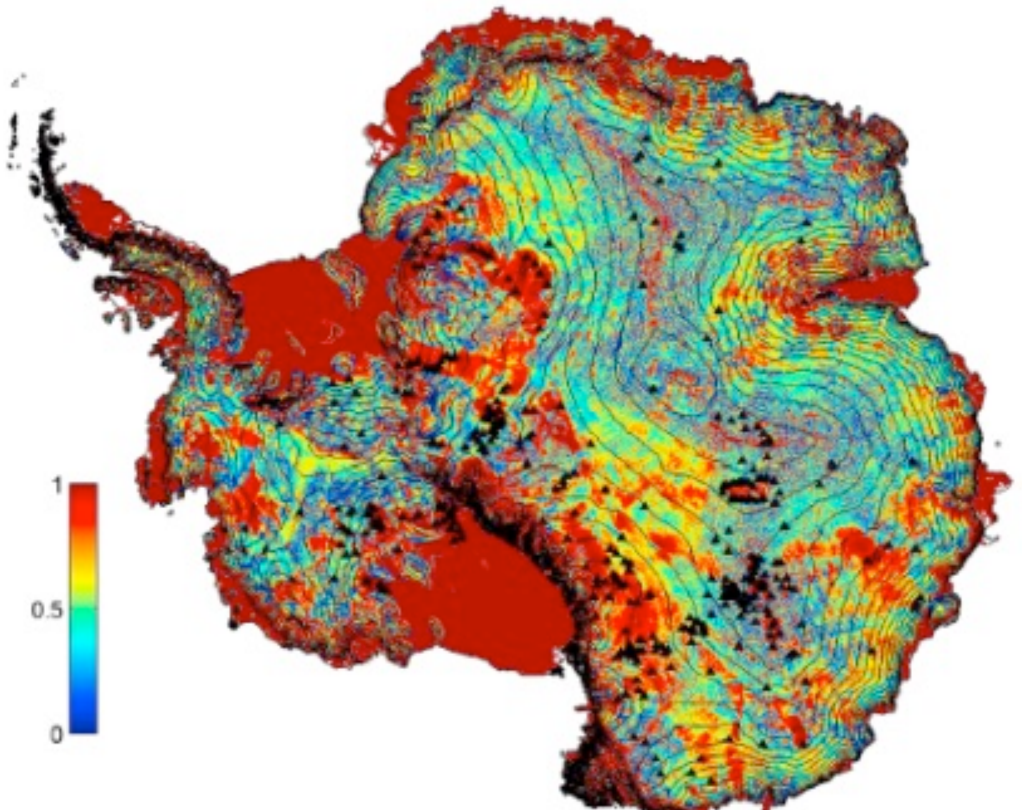


Figure 11. The ratio of flow speed at the bed to flow speed at the surface of the ice sheet.

Arthern et al. (2015)

What controls basal slip?

What controls basal slip?

$$u_b = \frac{\tau_b}{\psi}$$

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$$u_b = \frac{\tau_b}{\psi}$$

← Approx. equal to driving stress: $\tau_d = \rho g H \frac{\partial H}{\partial x}$

What controls basal slip?

$$u_b = \frac{\tau_b}{\psi}$$

$\tau_b = 120 \text{ kPa}$
$u_b = 6 \text{ m yr}^{-1}$

$$\psi = 20 \ \frac{\text{kP}a}{\text{m yr}^{-1}}$$

Glacier d' Arolla, Switzerland

What controls basal slip?

$$u_b = \frac{\tau_b}{\psi}$$

Variegated Glacier, Alaska

$\tau_b = 130\ \text{kPa}$
$u_b = 90\ \text{m yr}^{-1}$

$$\psi = 1\ \frac{\text{kP}a}{\text{m yr}^{-1}}$$

Austin Post, 1965

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$\tau_b = 130 \text{ kPa}$
$u_b = 4800 \text{ m yr}^{-1}$

$$\psi = 0.03 \ \frac{\text{kP}a}{\text{m yr}^{-1}}$$

Austin Post, 1965

What controls basal slip?

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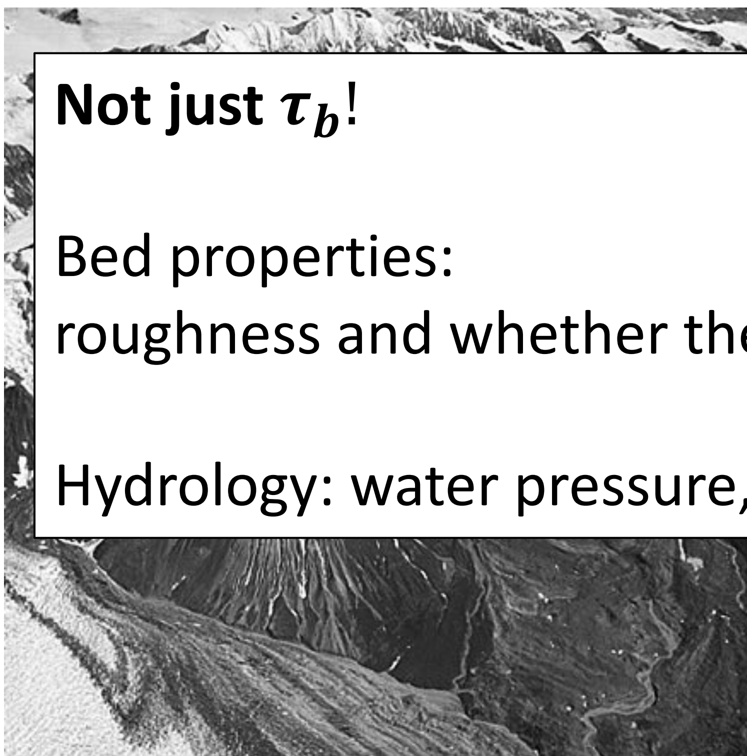
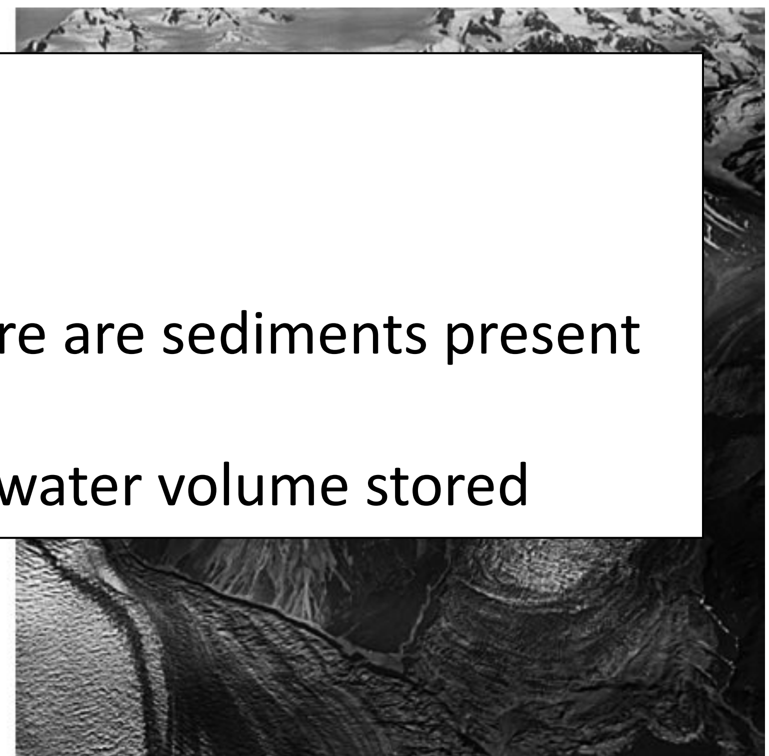




Not just $\boldsymbol{\tau_b}$!

Bed properties:
roughness and whether there are sediments present

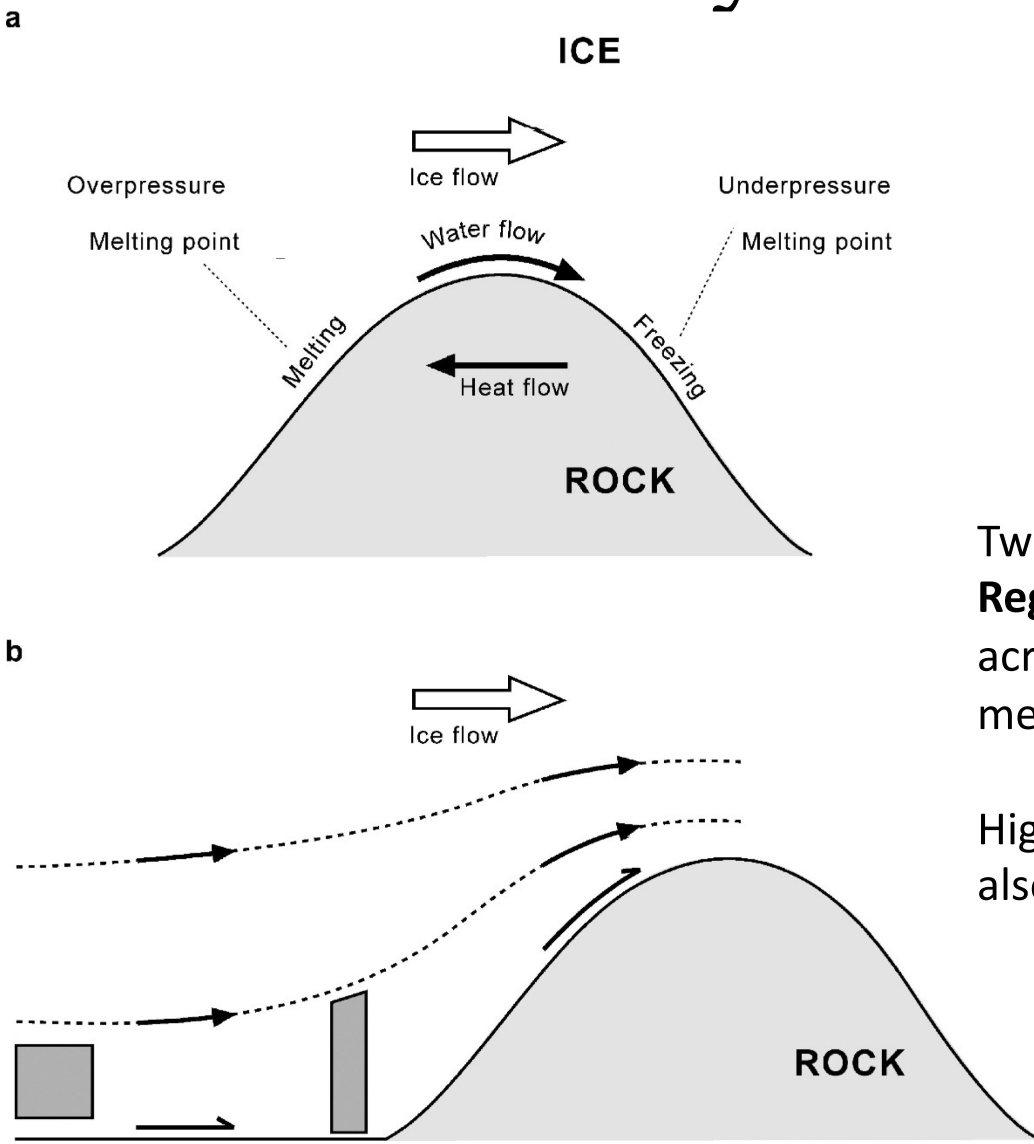
Hydrology: water pressure, water volume stored

Austin Post, 1965

Hard glacier beds

Weertman’s theory for hard beds

Weertman's theory for hard beds



- Assume we are at the melting point everywhere.
- Assume no cavitation
- Assume thin water film at ice rock interface.

Two mechanisms:
Regelation: temperature gradient across bumps causes heat flow and melting/re-freezing

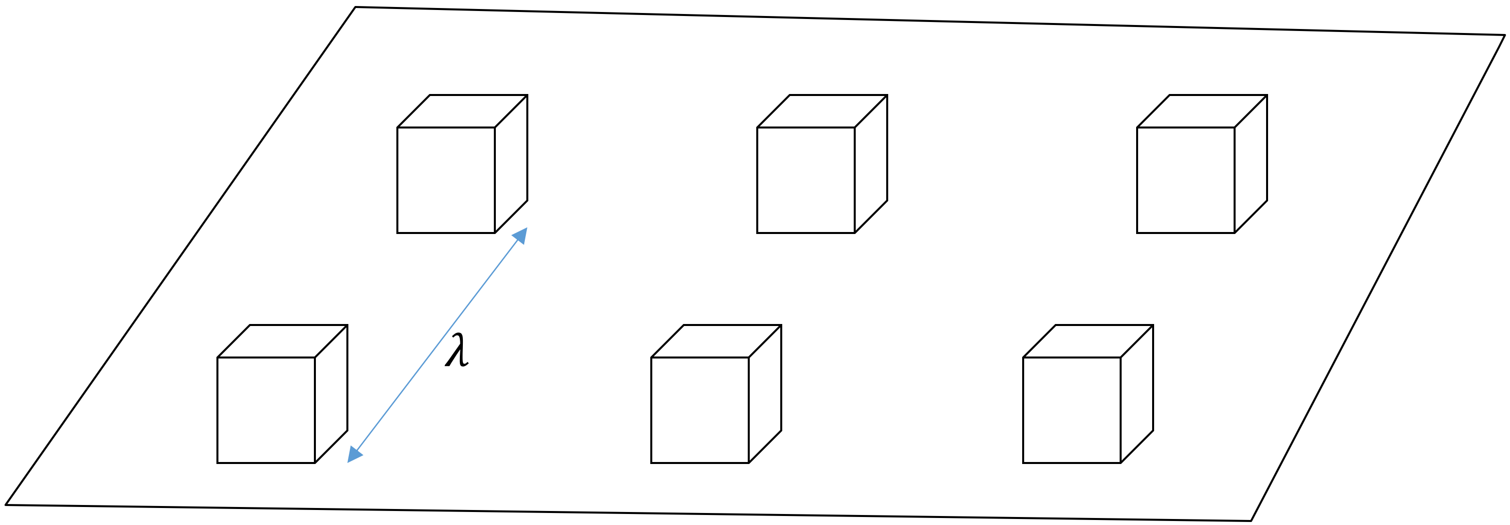
Higher stress on the upstream side also causes **enhanced creep**.

Regelation demo:

https://www.youtube.com/watch?v=qQCVnjGUv24

Weertman's theory for hard beds

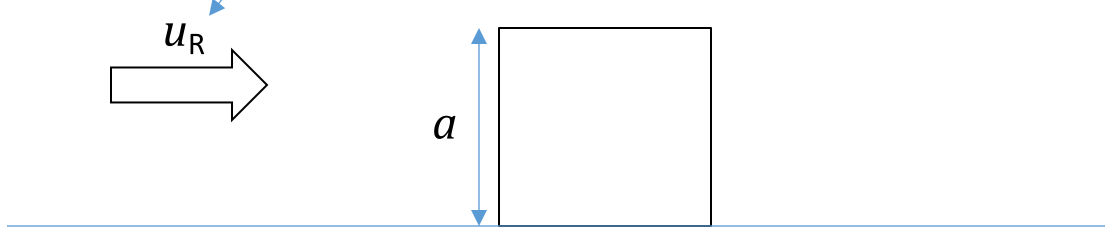
Basal shear stress: τ_b



Force on each cube: $\tau_b \lambda^2$

Weertman's theory for hard beds

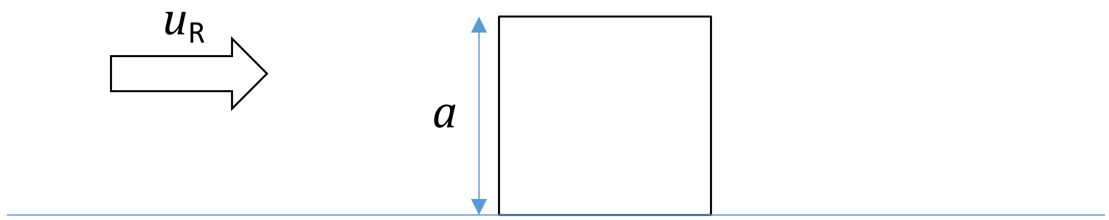
Sliding speed due to regelation



Regelation

Force on each cube = $\tau_b \lambda^2$

Weertman's theory for hard beds

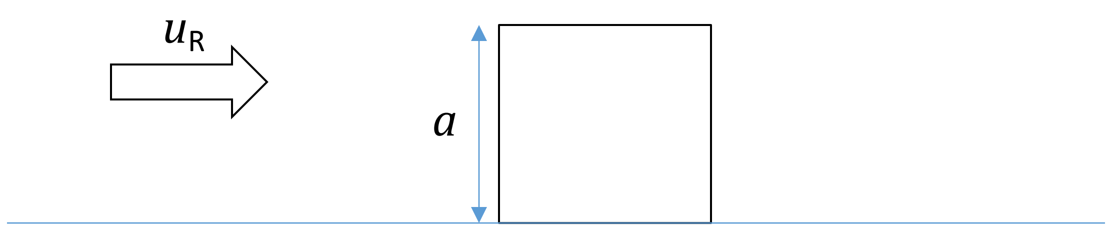


Regelation

Force on each cube = $\tau_b \lambda^2$

Force on each side of the cube = $\frac{1}{2}\tau_b \lambda^2$

Weertman's theory for hard beds



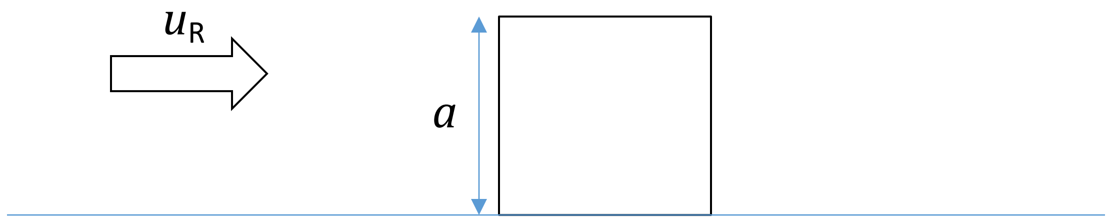
Regelation

Force on each cube = $\tau_b \lambda^2$

Force on each side of the cube = $\frac{1}{2}\tau_b \lambda^2$

Stress on each side of the cube = $\frac{1}{2}\tau_b \left(\frac{\lambda}{a}\right)^2$

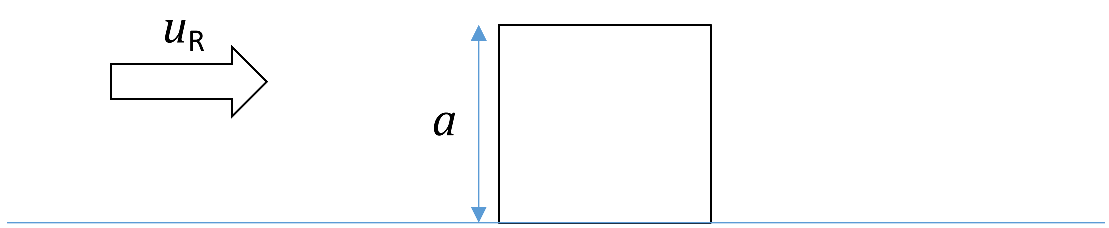
Weertman's theory for hard beds



Regelation

Total differential stress across each cube = $\tau_b \left(\frac{\lambda}{a}\right)^2$

Weertman's theory for hard beds



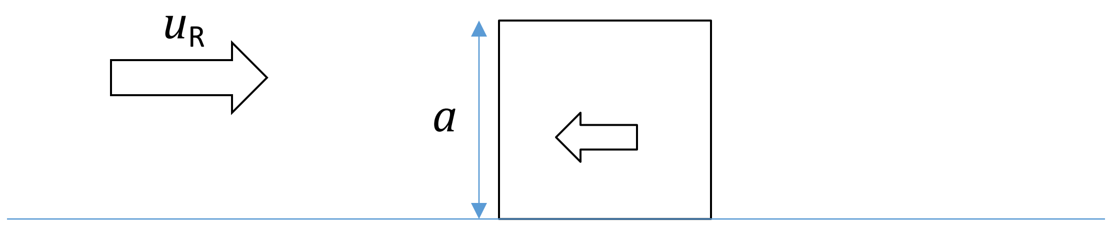
Regelation

$\mathcal{B}$ is the dependence of the melting point of water on pressure.
= 9.8x10^{-5} K kPa^{-1}
(= 8.7x10^{-4} Km^{-1} in ice)

Total differential stress across each cube = $\tau_b \left(\frac{\lambda}{a}\right)^2$

Melting point is a function of stress, so difference in T across the cube is $\delta T = \mathcal{B}\tau_b \left(\frac{\lambda}{a}\right)^2$

Weertman's theory for hard beds



Regelation

k is thermal conductivity of the rock.

…and heat flow through the cube = $ak\mathcal{B}\tau_b\left(\frac{\lambda}{a}\right)^2$

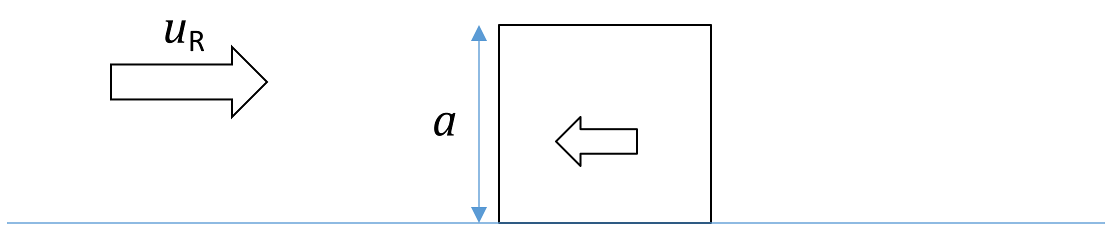
a^2 = Cross-sectional area

Heat flux per unit cross-sectional area, $q = k\frac{dT}{dx}$, so total heat flux is

$$k\frac{a^2\delta T}{a} = k\delta Ta = ak\mathcal{B}\tau_b\left(\frac{\lambda}{a}\right)^2$$

Distance over which temperature gradient acts

Weertman's theory for hard beds



Regelation

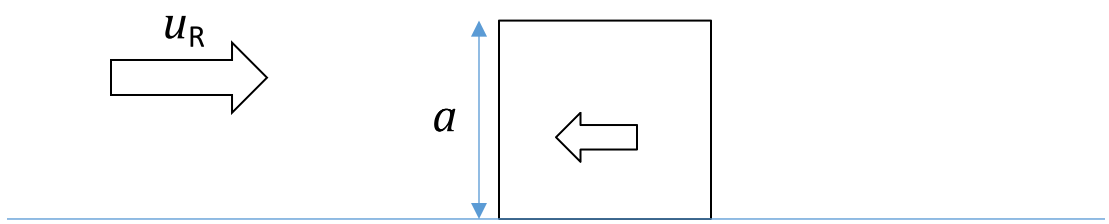
L is the latent heat of fusion of water.

…and heat flow through the cube = $ak\mathcal{B}\tau_b\left(\frac{\lambda}{a}\right)^2$

Volume of ice melted per unit time due to this heat is

$$\dot{m} = \frac{ak\mathcal{B}\tau_b}{\rho_i L}\left(\frac{\lambda}{a}\right)^2$$

Weertman's theory for hard beds



Regelation

…and heat flow through the cube = $ak\mathcal{B}\tau_b\left(\frac{\lambda}{a}\right)^2$

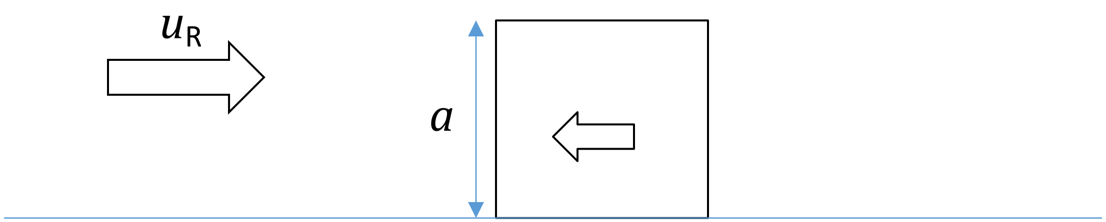
Volume of ice melted per unit time due to this heat is

$$\dot{m} = \frac{ak\mathcal{B}\tau_b}{\rho_i L}\left(\frac{\lambda}{a}\right)^2$$

But flow rate is controlled by the melt rate so $\dot{m} = u_R a^2$

Weertman's theory for hard beds

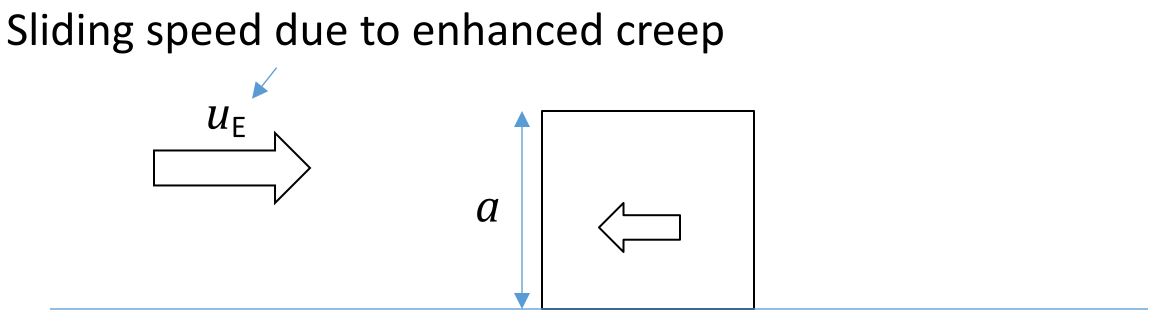
Regelation



$$u_R a^2 = \frac{ak\mathcal{B}\tau_b}{\rho_i L}\left(\frac{\lambda}{a}\right)^2$$

$$\boxed{u_R = \frac{k\mathcal{B}\tau_b}{a\rho_i L}\left(\frac{\lambda}{a}\right)^2}$$

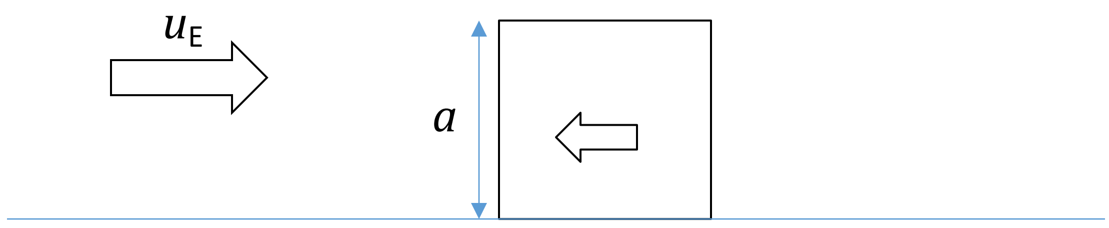
Weertman's theory for hard beds



Enhanced Creep

Stress on each side of the cube = $\frac{1}{2}\tau_b\left(\frac{\lambda}{a}\right)^2$

Weertman's theory for hard beds

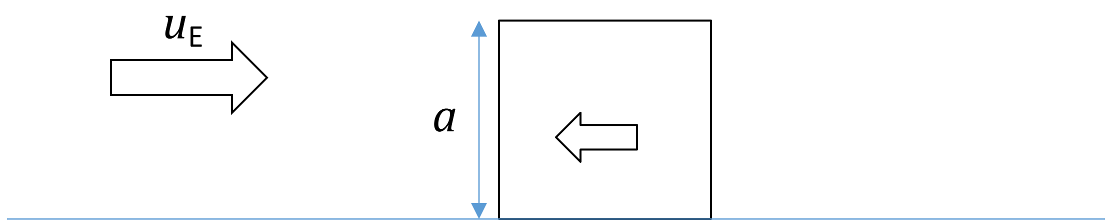


Enhanced Creep

Stress on each side of the cube = $\frac{1}{2}\tau_b\left(\frac{\lambda}{a}\right)^2$

Generates a strain rate $\propto A\left[\tau_b\left(\frac{\lambda}{a}\right)^2\right]^n$

Weertman's theory for hard beds



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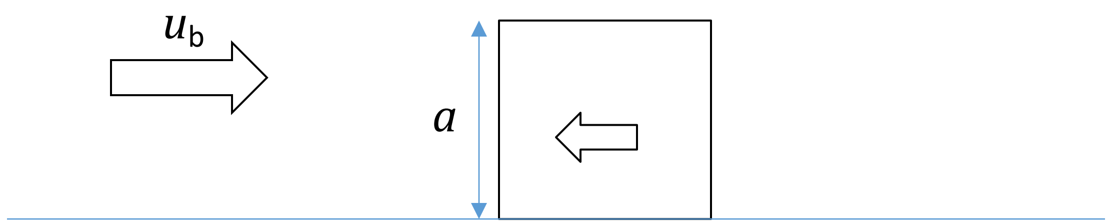
Generates a strain rate $\propto A\left[\tau_b\left(\frac{\lambda}{a}\right)^2\right]^n$

We assume the strain occurs over a length scale a, so:

$$u_E \propto aA\left[\tau_b\left(\frac{\lambda}{a}\right)^2\right]^n$$

Weertman's theory for hard beds



Regelation

$$u_R = \frac{k\mathcal{B}\tau_b}{a\rho_i L}\left(\frac{\lambda}{a}\right)^2$$

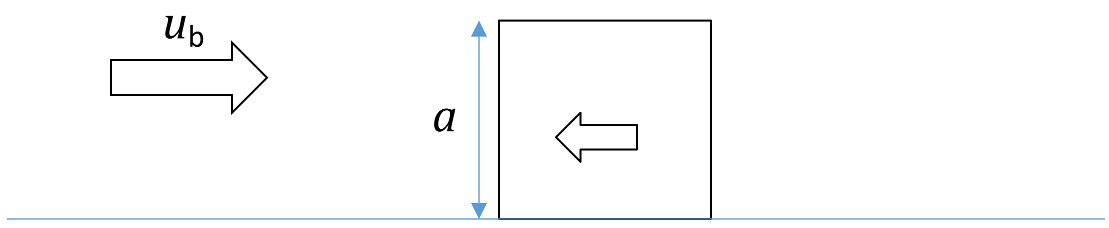
Enhanced Creep

$$u_E \propto aA\left[\tau_b\left(\frac{\lambda}{a}\right)^2\right]^n$$

For a given obstacle size: $u_b = \max(u_R, u_E)$

But there is a whole range of obstacle sizes,
so one obstacle size controls the sliding speed

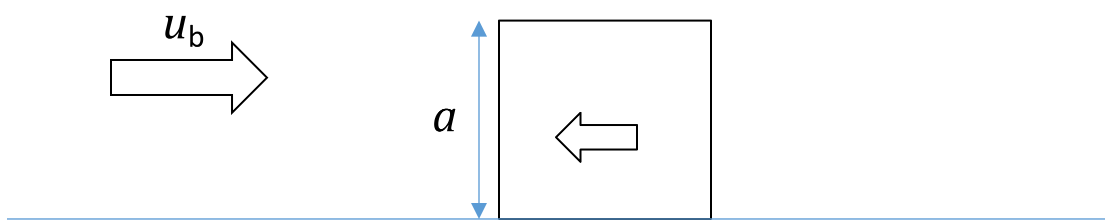
Weertman's theory for hard beds



Equate these and re-arrange for the critical obstacle size, a_c:

$$a^2 \propto \frac{k\mathcal{B}\tau_b^{1-n}R^{2(n-1)}}{\rho_i LA}$$

Weertman's theory for hard beds



$$u_R = \frac{k\mathcal{B}\tau_b}{a\rho_i L}\left(\frac{\lambda}{a}\right)^2$$

Equal these and re-arrange for the critical obstacle size, a_c:

$$a^2 \propto \frac{k\mathcal{B}\tau_b^{1-n} R^{2(n-1)}}{\rho_i L A}$$

Substituting back into the regelation equation (above) gives:

$$u_b \propto \left(\frac{\sqrt{\tau_b}}{R}\right)^{n+1}$$

$$R = \frac{\lambda}{a}$$

Assumptions are restrictive and this is a crude model, but it captures some essence of the processes.

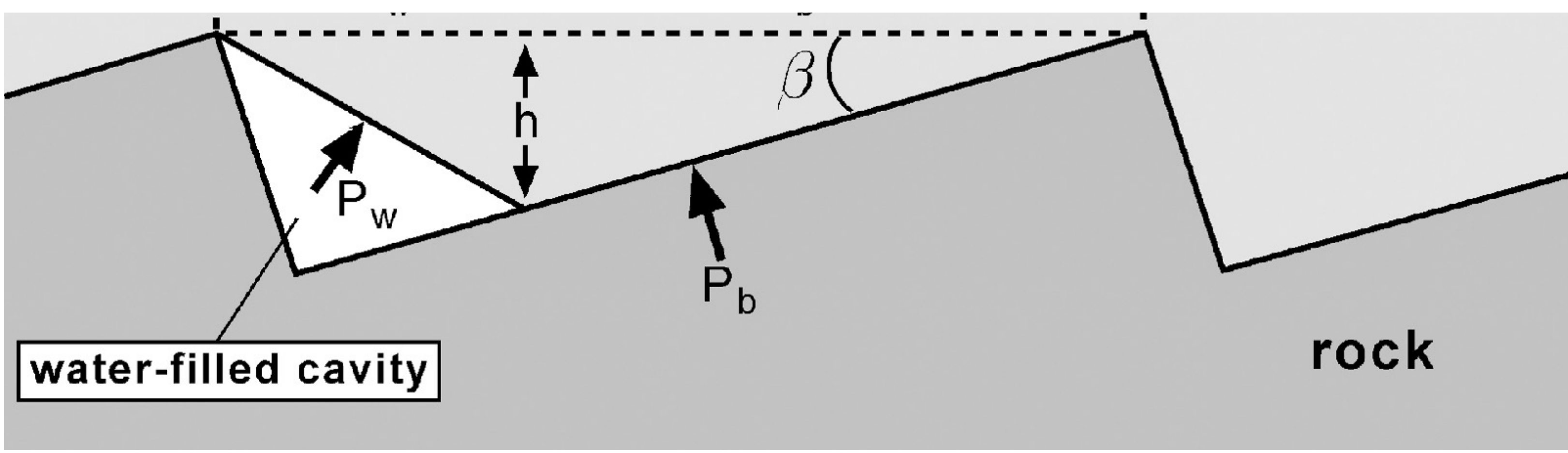
Cavitation

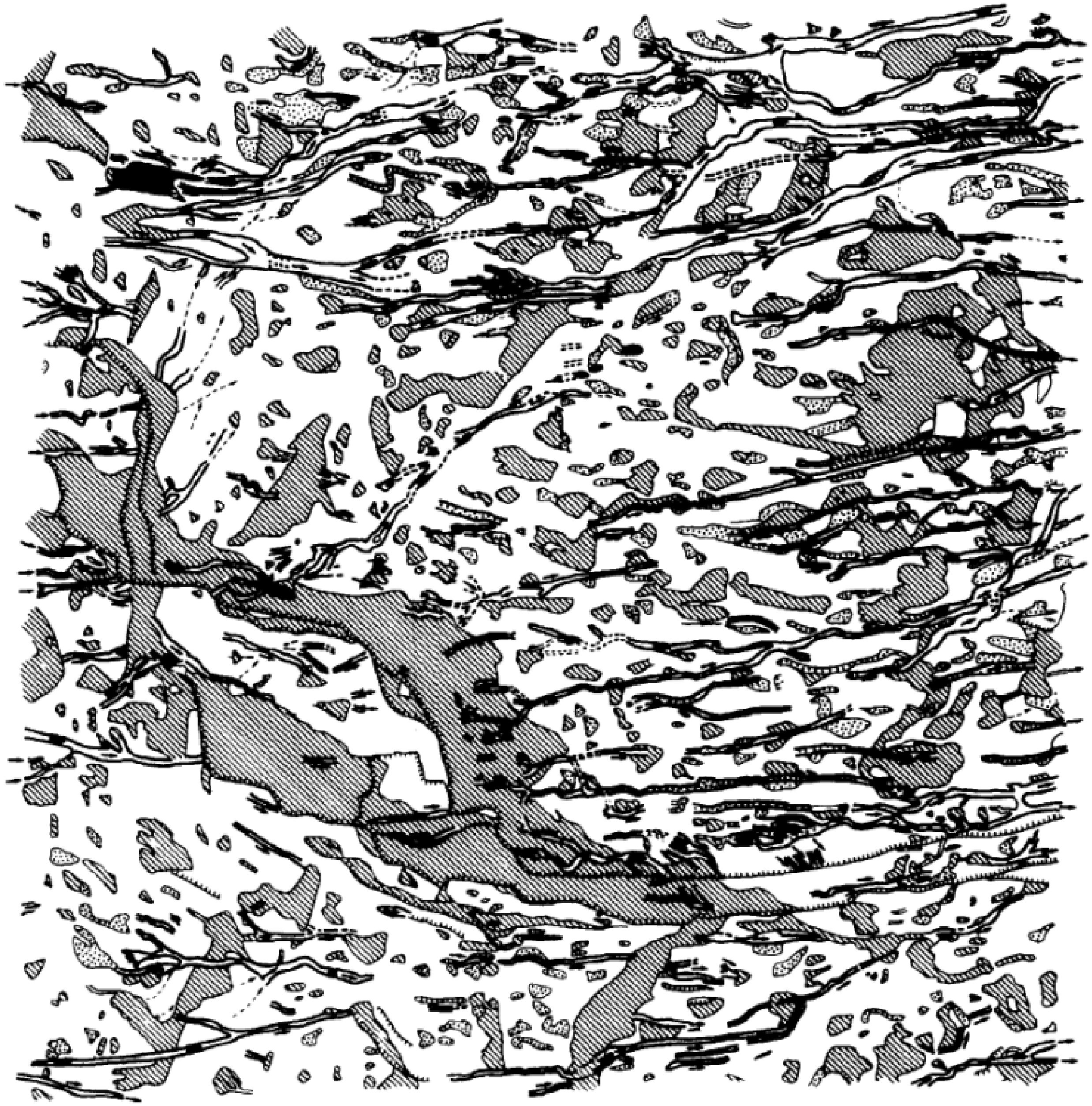
N is the effective pressure = p_i - p_w

$$u_b \propto \frac{\tau_b^p}{RN^q}$$

Where water pressure is high enough, water filled cavities form.

This increases sliding….



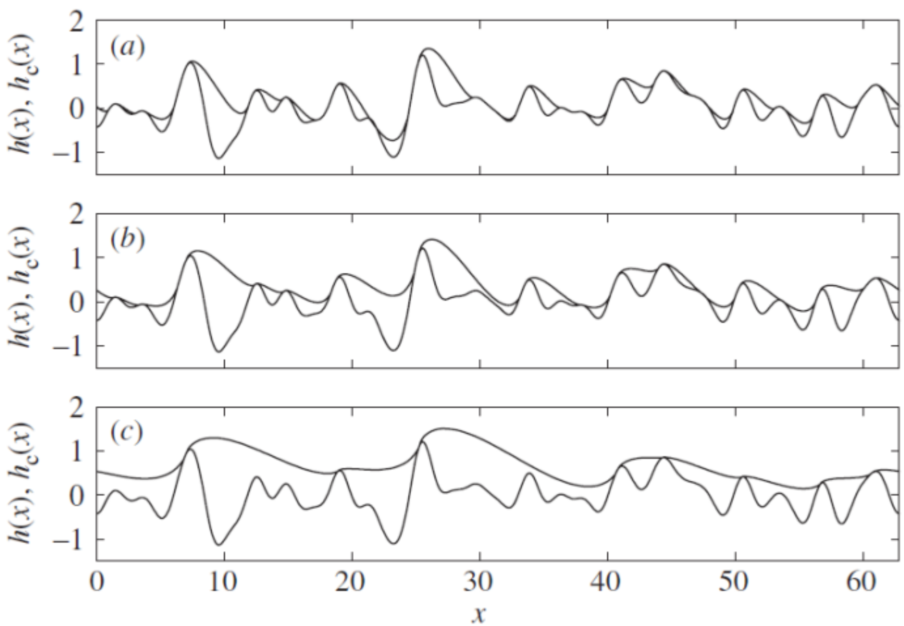


Sharp et al, (1989)

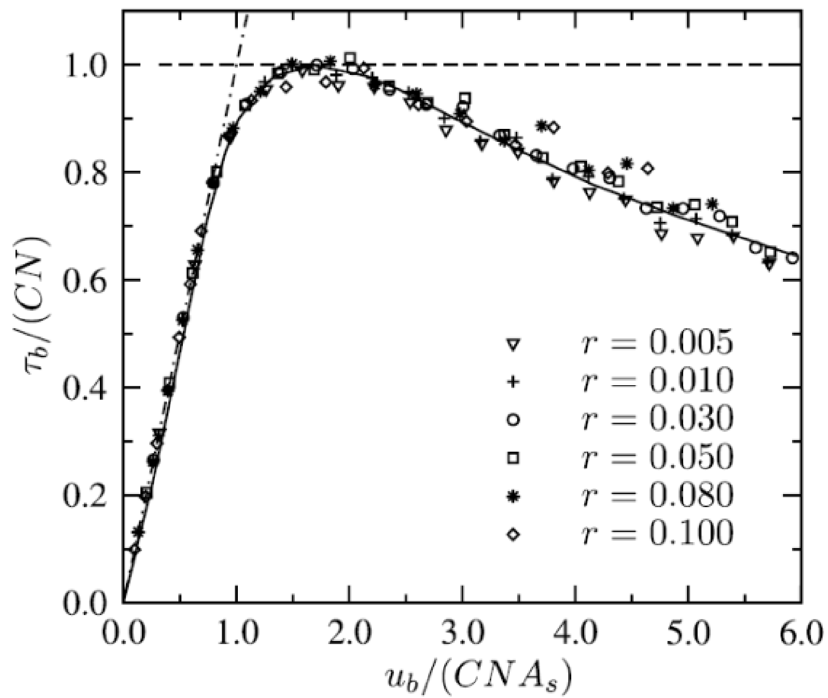
Iken’s bound

Iken (1981)

Shear stress generated at the bed cannot exceed some bound because cavities grow so much that they start drowning smaller ones downstream



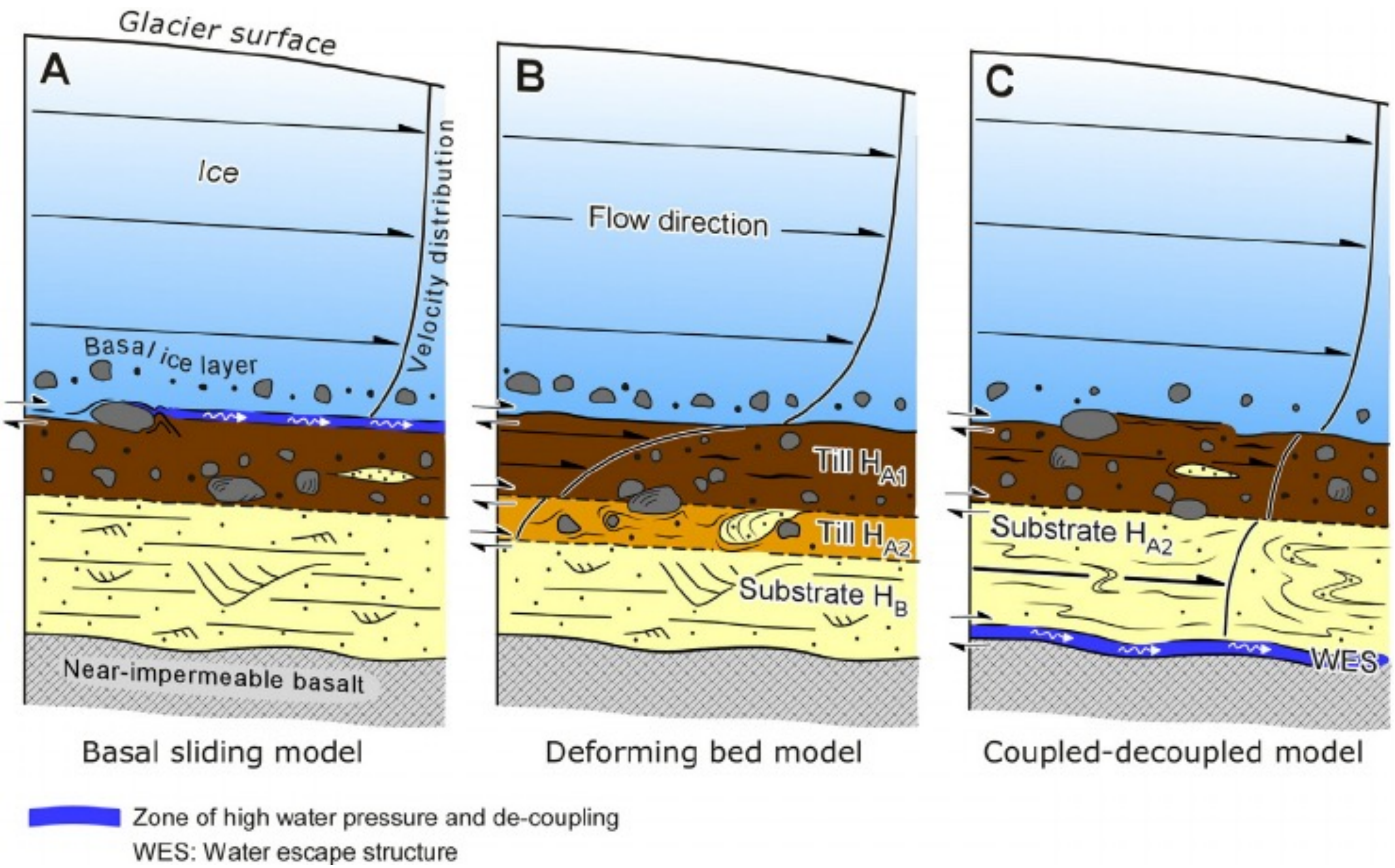
Schoof (2005)



Gagliardini et al. (2007)

"Soft beds":

deformable material under the ice.



Modified from Kjær et al. (2006)

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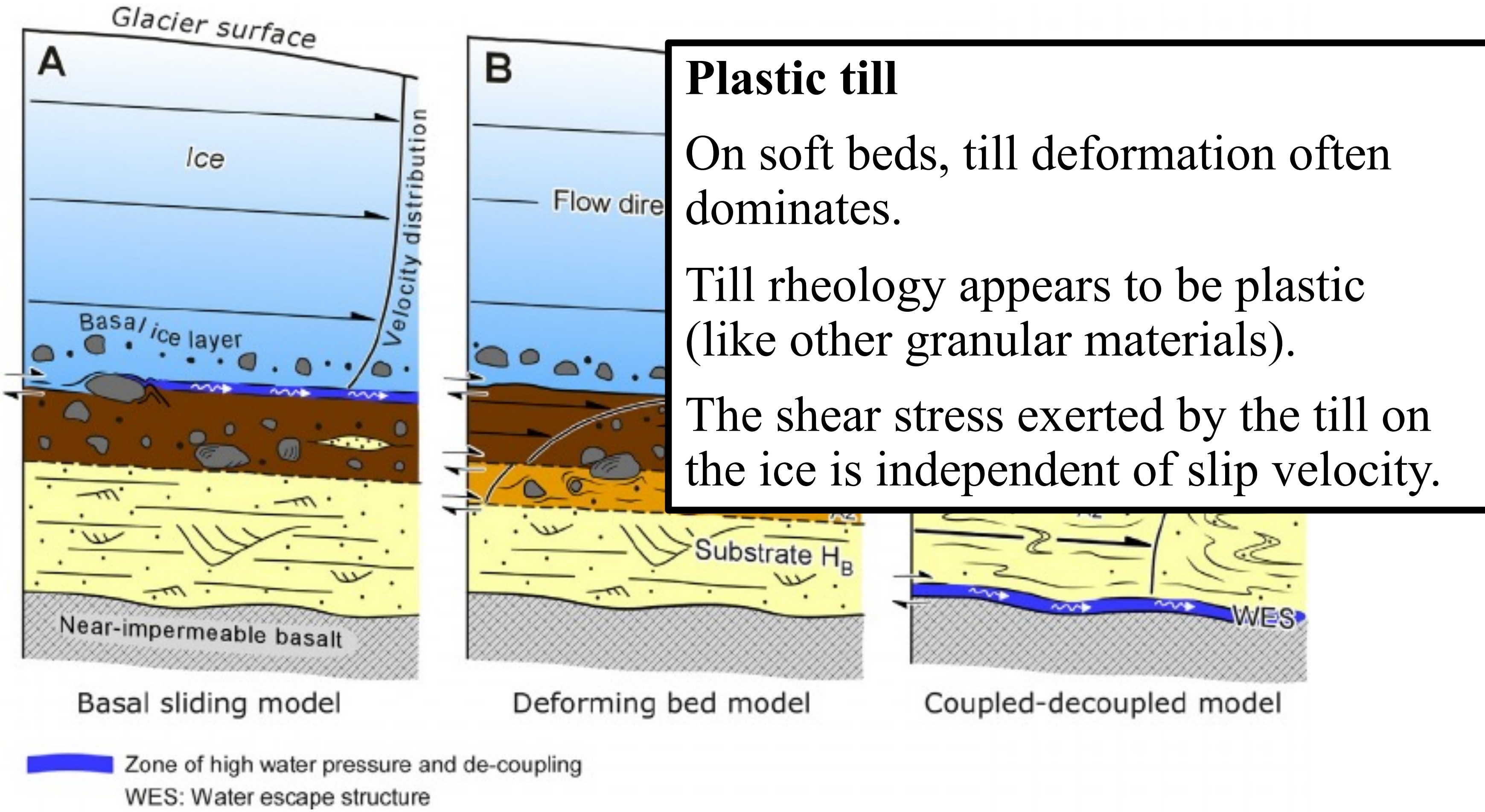


Plastic till

On soft beds, till deformation often dominates.

Till rheology appears to be plastic (like other granular materials).

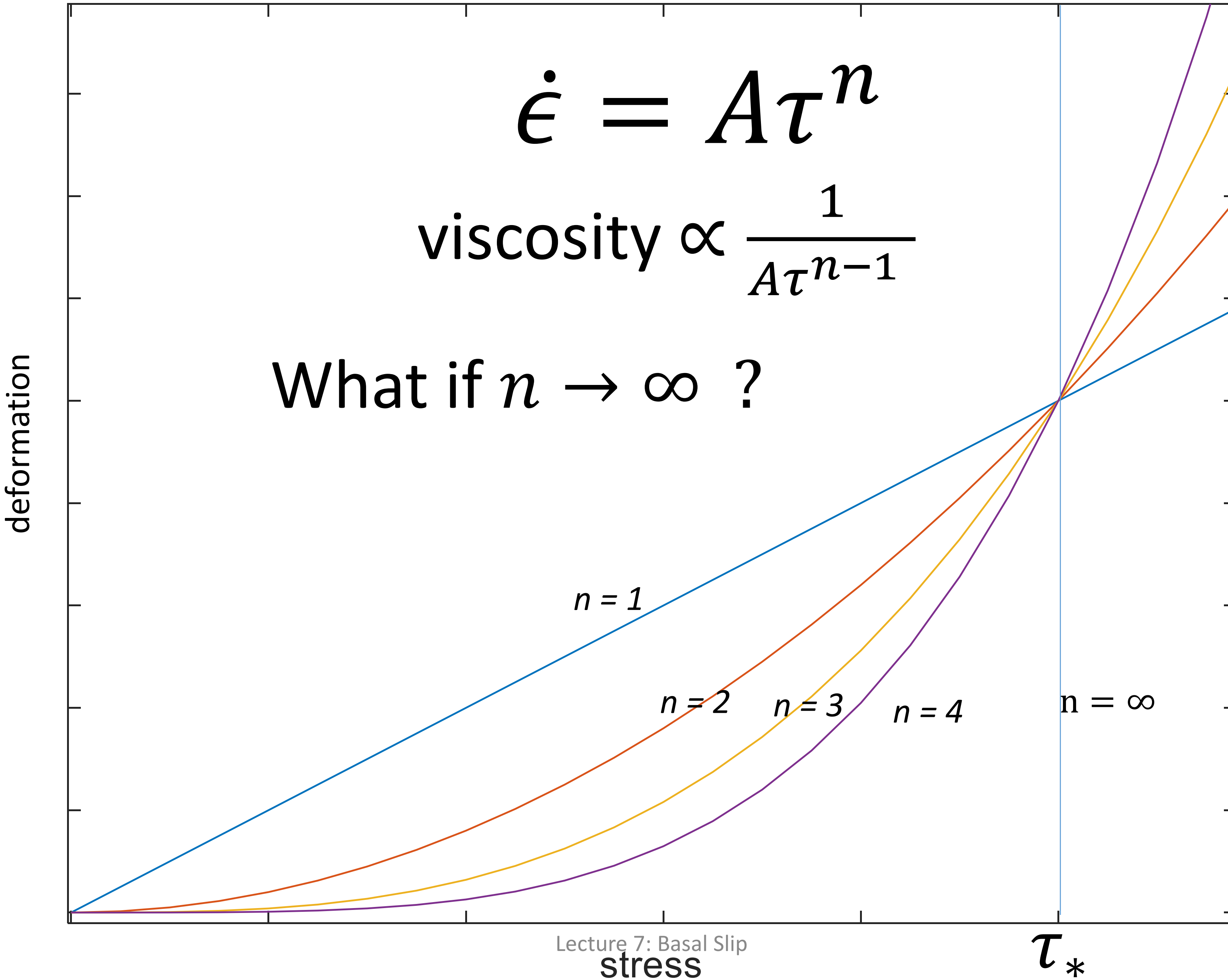
The shear stress exerted by the till on the ice is independent of slip velocity.

Modified from Kjær et al. (2006)

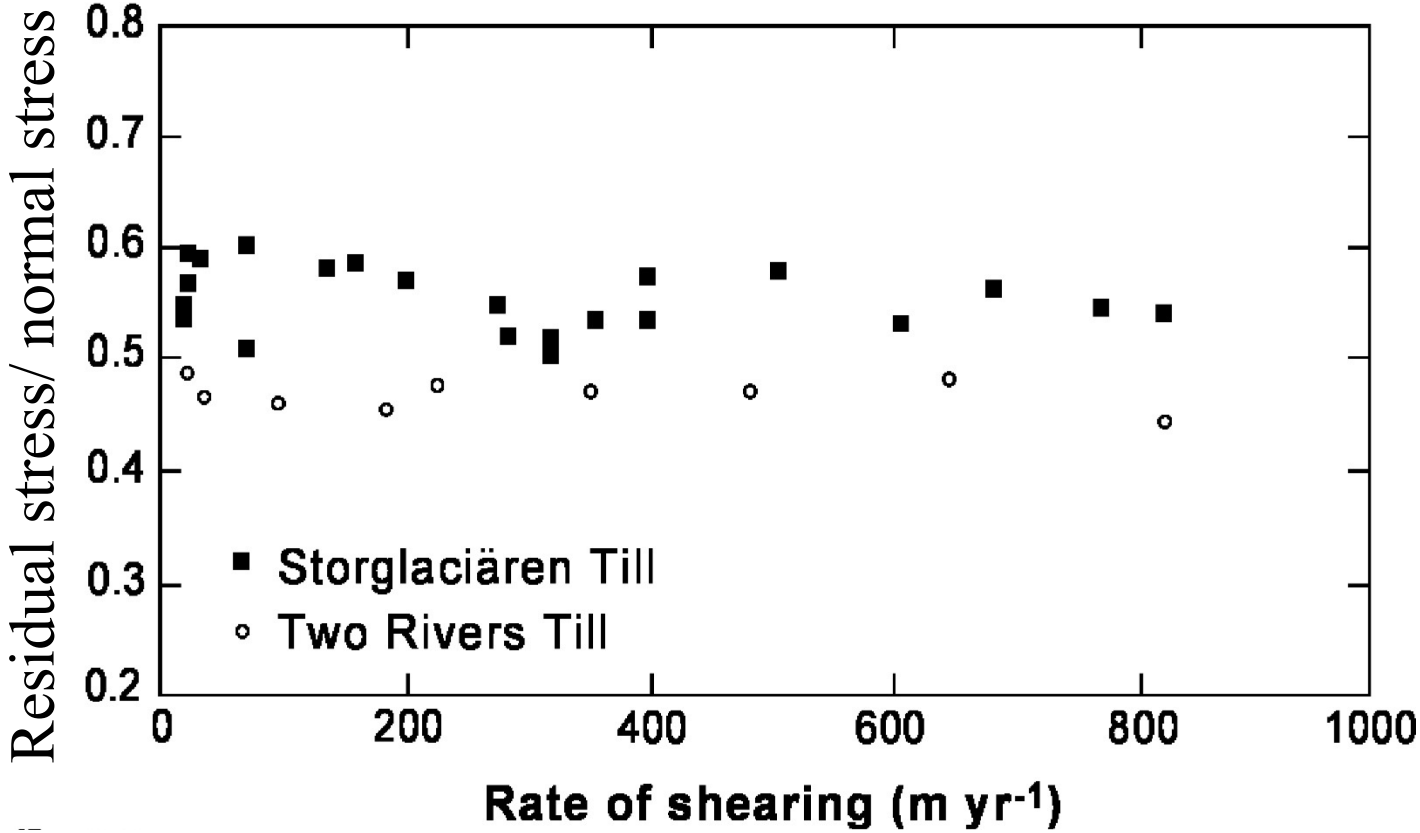
$$\dot{\epsilon} = A\tau^n$$

$$\text{viscosity} \propto \frac{1}{A\tau^{n-1}}$$

What if $n \to \infty$?



Mohr-Coulomb sliding



Cuffey and Paterson

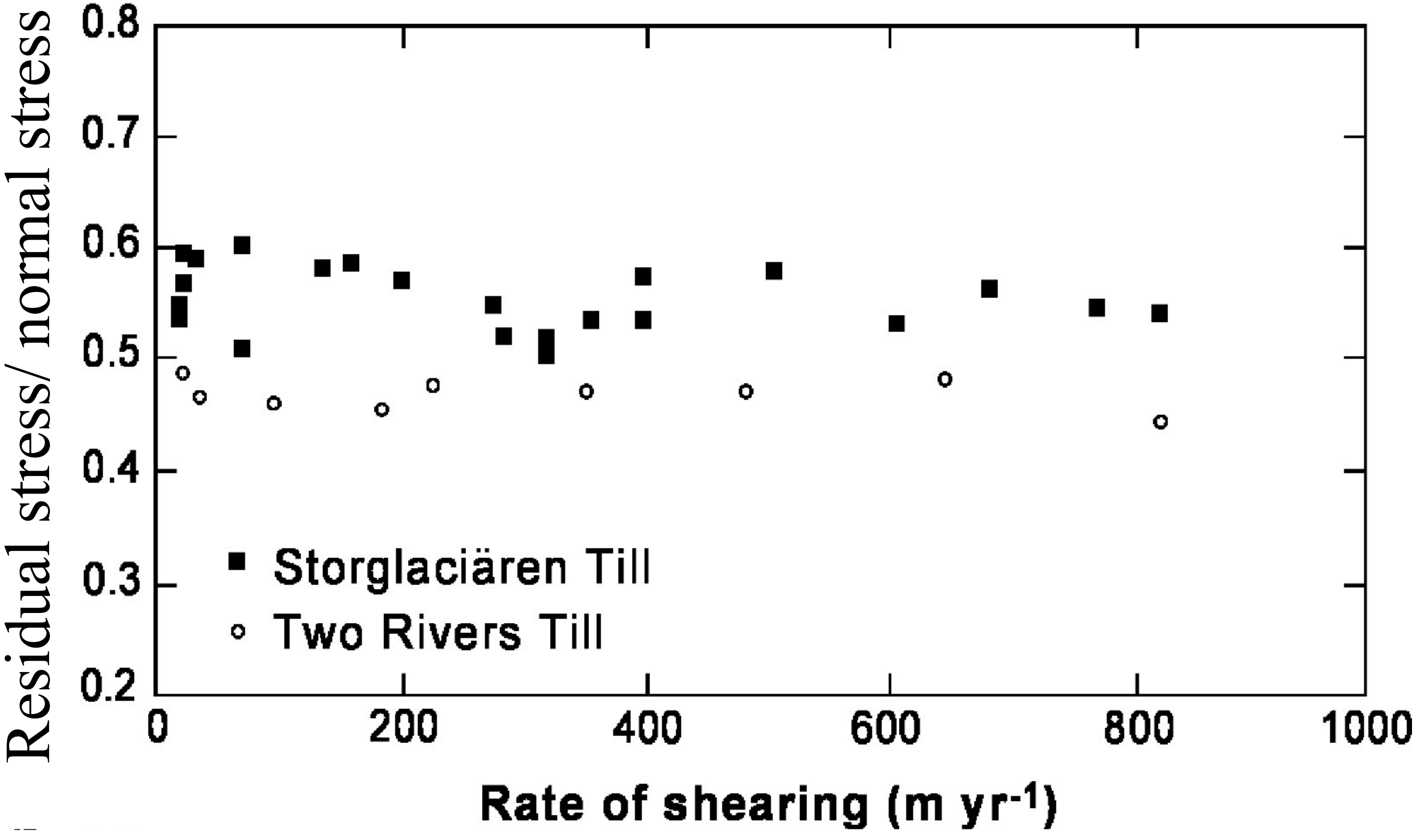
Mohr-Coulomb sliding

Residual stress

$$\tau_* = c_0 + fN$$

Effective pressure

$$N = P_i - P_w$$



Cuffey and Paterson

Mohr-Coulomb sliding

Residual stress

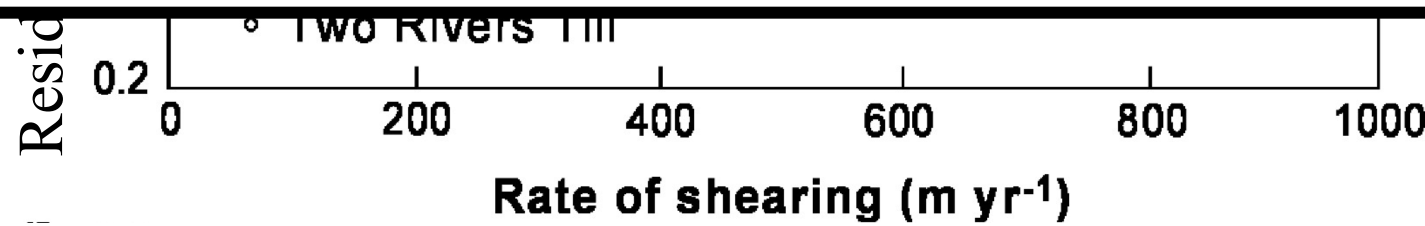
$$\tau_* = c_0 + fN$$

Effective pressure

$$N = P_i - P_w$$

Very different behavior than "Weertman-style" sliding.

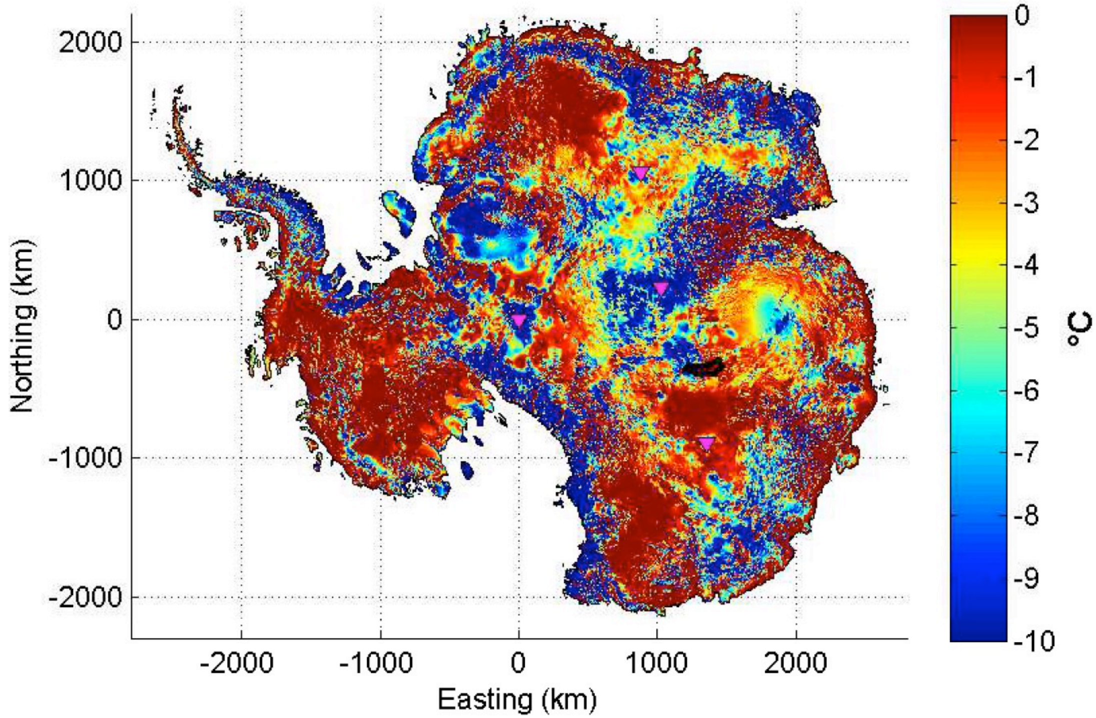
Much current research is aimed at understanding which of these are 'correct'.



Cuffey and Paterson

Implications

- Temperature at the base controls ice flux by determining if slip occurs.
- Water at the bed controls ice flux by determining how fast slip occurs.
- Glaciers erode and move sediment, modifying the landscape.



Van Liefferinge and Pattyn (2013)

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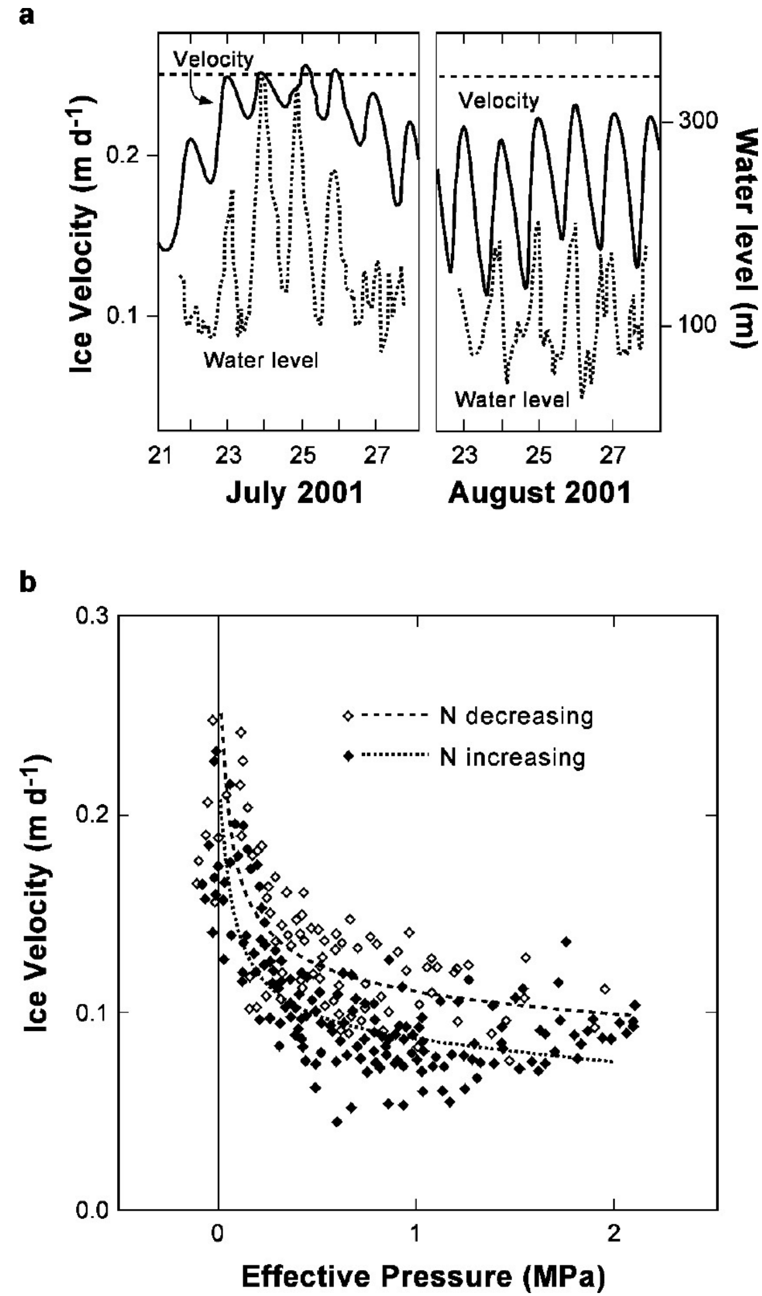

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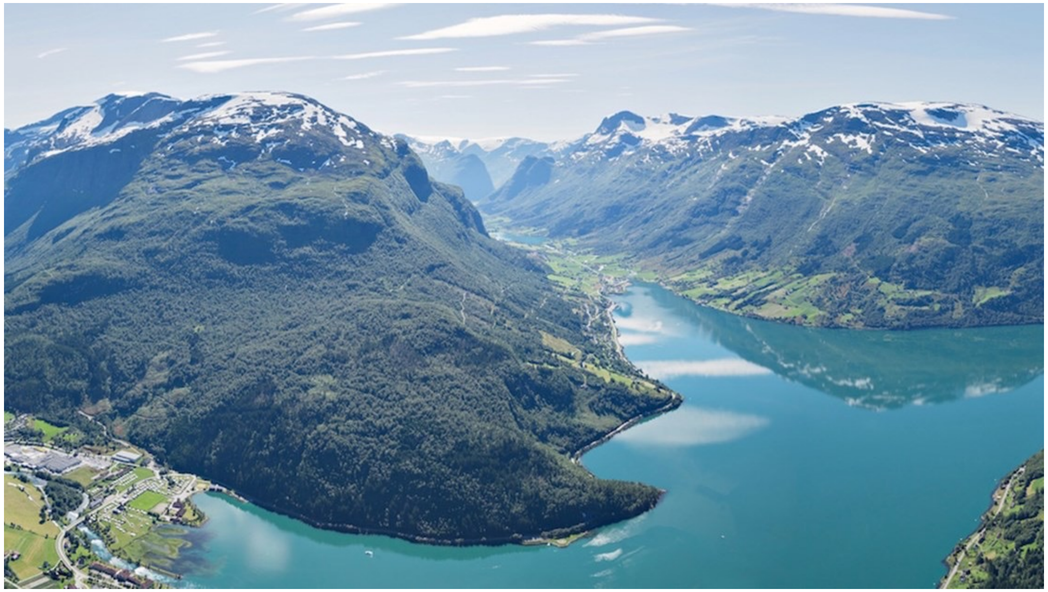

Major questions remain

- Which areas of the bed behave plastically and which behave like Weertman's model? (If these models are even good descriptions of sliding)
- How are water pressures and sliding speed coupled, quantitatively?
- How will the sliding relation evolve in time as ice sheets accelerate and thin?

Summary

- Glaciers slide over their beds.
- Over hard beds, without cavitation, Weertman's sliding model predicts sliding depends on ${\tau_b}^{\frac{n+1}{2}}$
- Cavitation increases sliding, introduces a dependence on water pressure and in theory can cause unstable sliding.
- Soft beds appear to be plastic, with the resistance provided by the bed not depending on sliding speed.

References

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