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# Physics Informed Neural Networks –
## for Identification and Forecasting of Chaotic Dynamics

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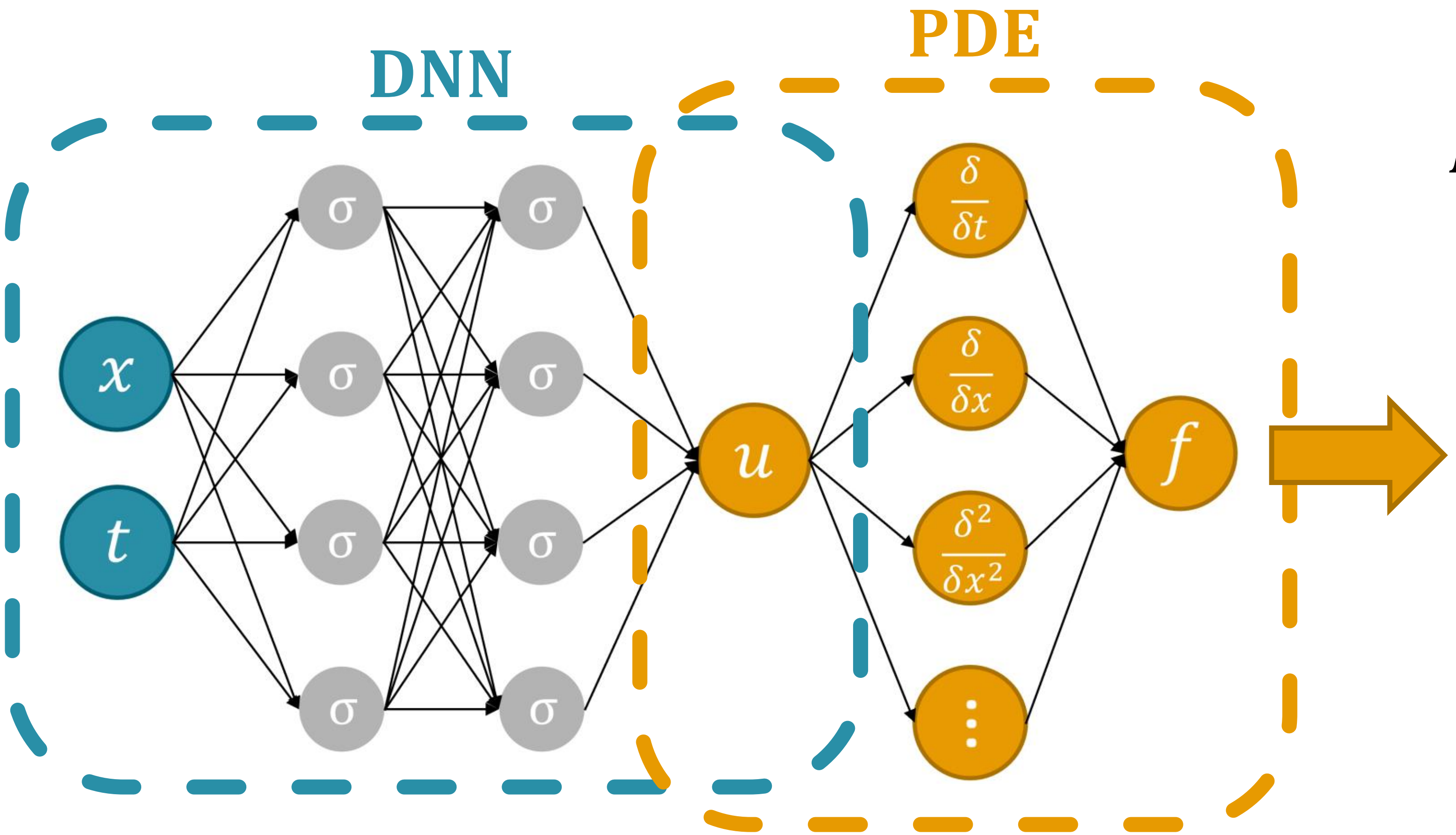
# Introduction

- Chaotic dynamics in nature

https://en.wikipedia.org/wiki/Chaos_theory

# Introduction

- Why physics informed neural networks?




$$PDE\colon u_t + \mathcal{N}[u] = 0$$
$$f := u_t + \mathcal{N}[u]$$

$$\mathcal{L} \coloneqq \mathcal{L}_u + \mathcal{L}_{i/b} + \mathcal{L}_f$$

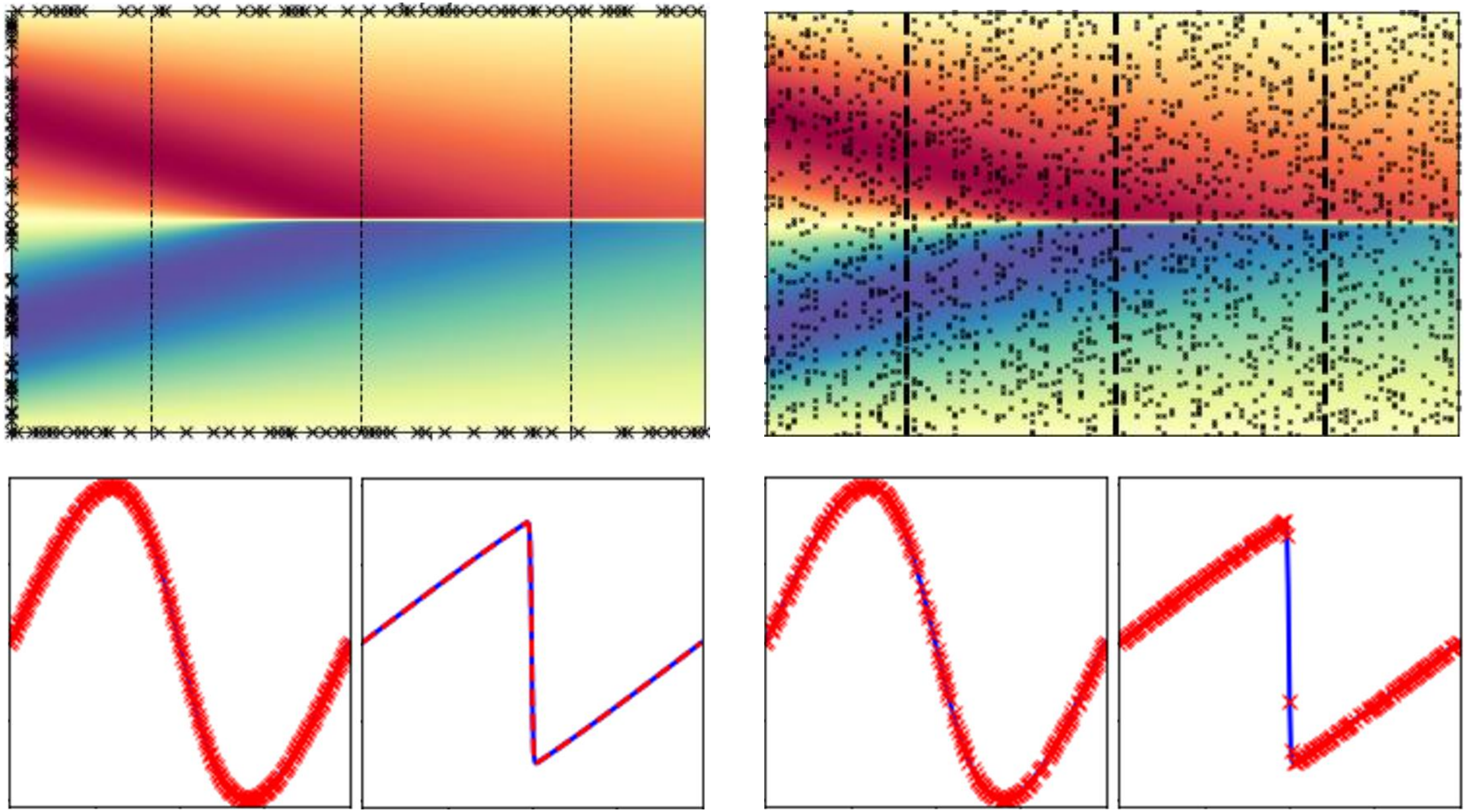
# Goal of Thesis

Inference

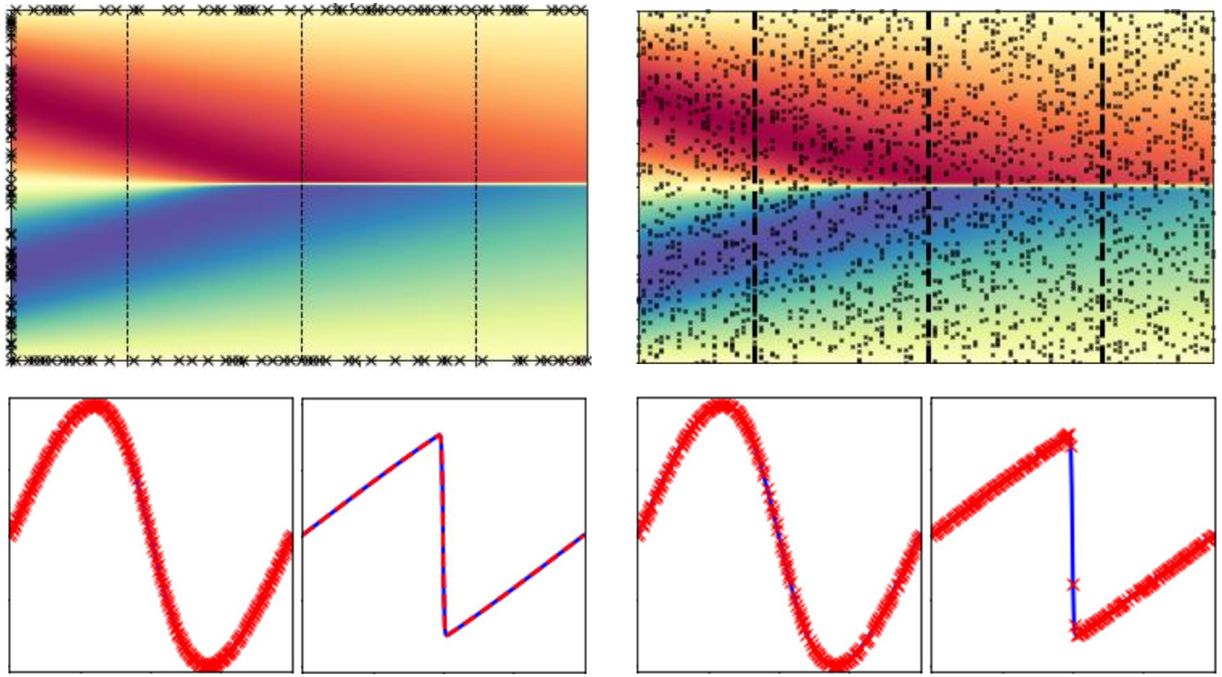
Identification

Continuous-time

Discrete-time



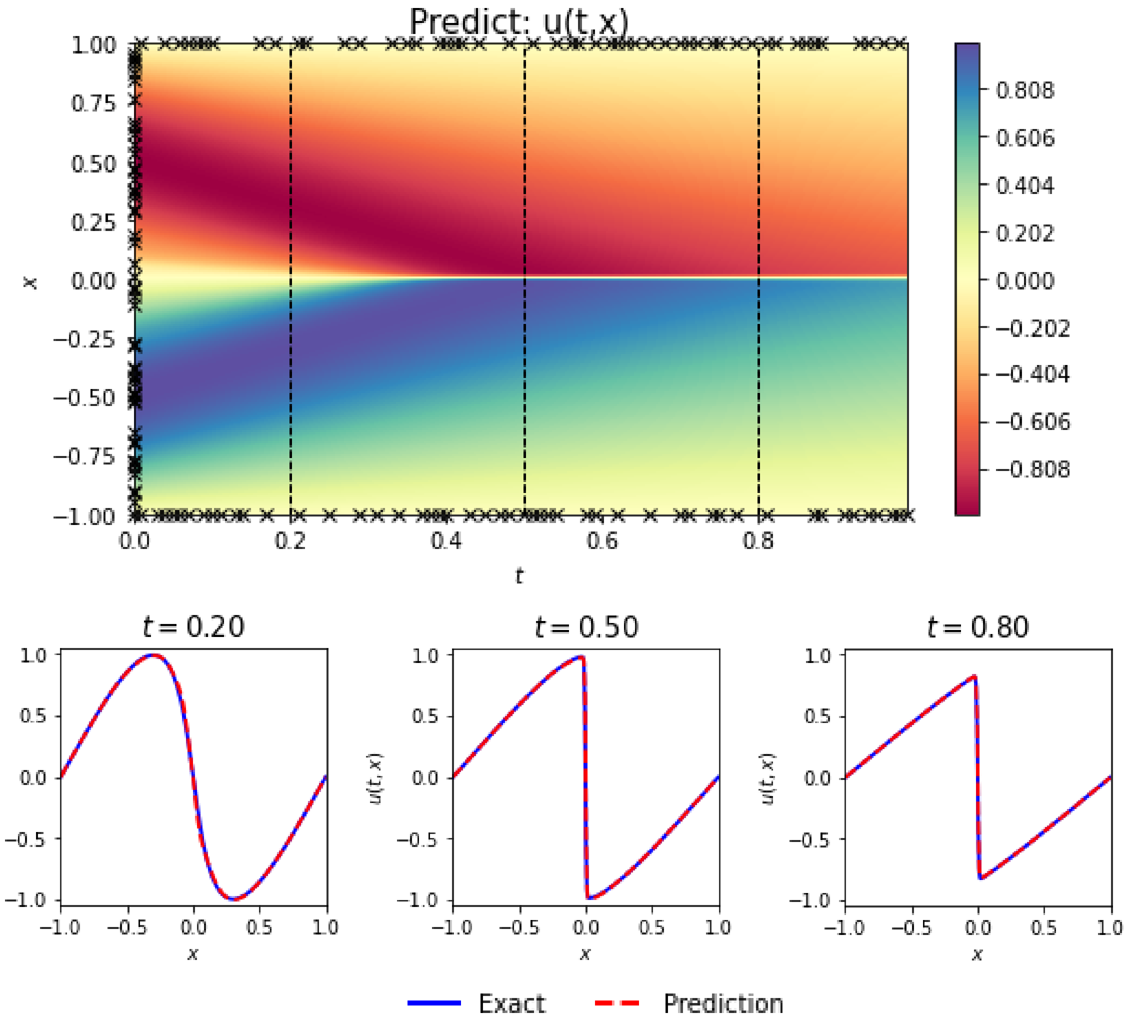
# Goal of Thesis



**Apply to more complex examples**
- Nonlinear Schrodinger equation
- Kuramoto-Sivashinsky equation

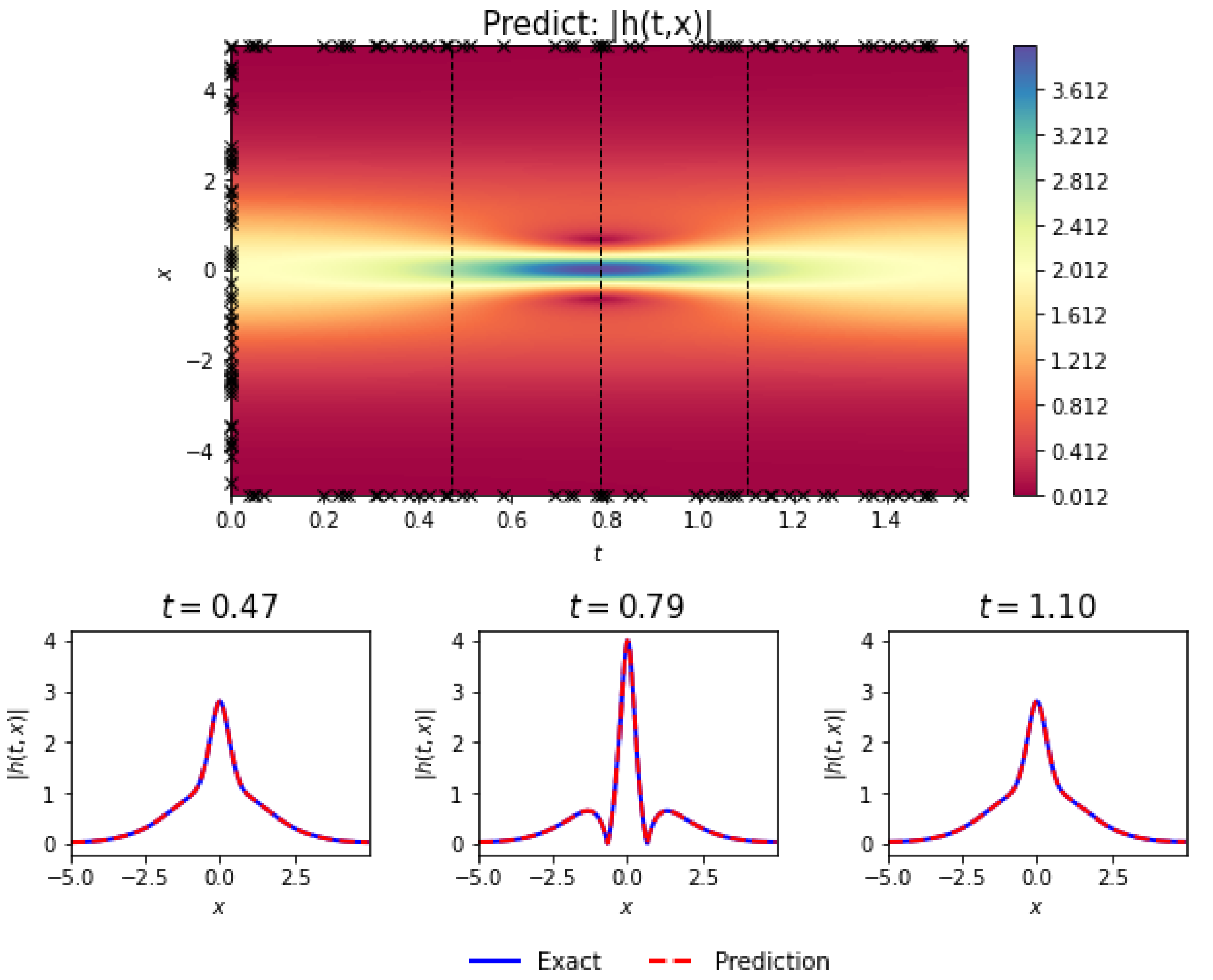
# Continuous-time Inference — The Burgers equation

$$u_t + uu_x - (0.01/\pi)u_{xx} = 0$$



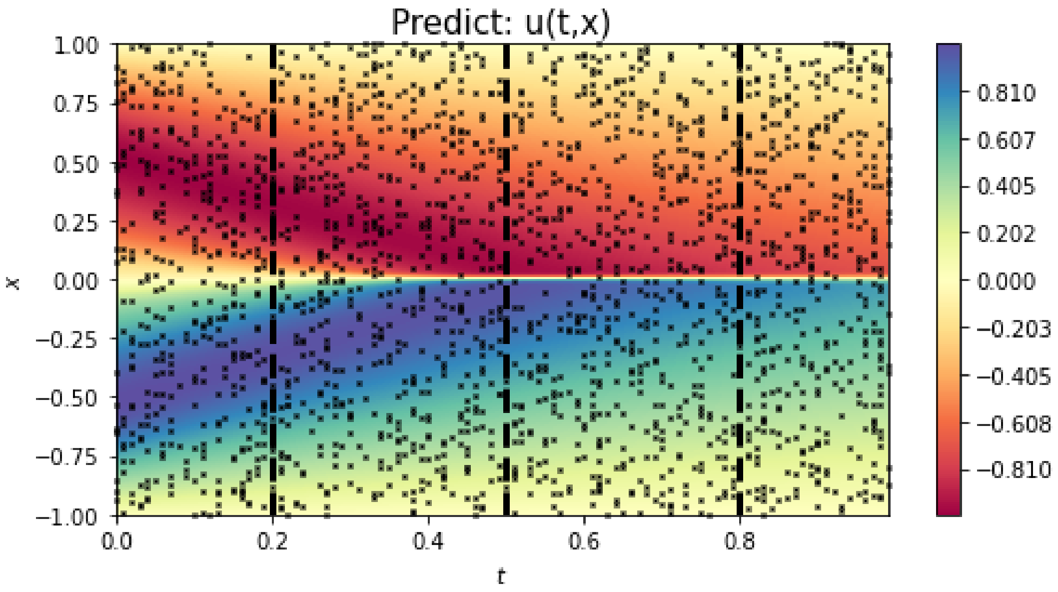
# → The Nonlinear Schrodinger equation

$$ih_t + 0.5h_{xx} + |h|^2 h = 0$$



# Continuous-time Identification — The Burgers Equation

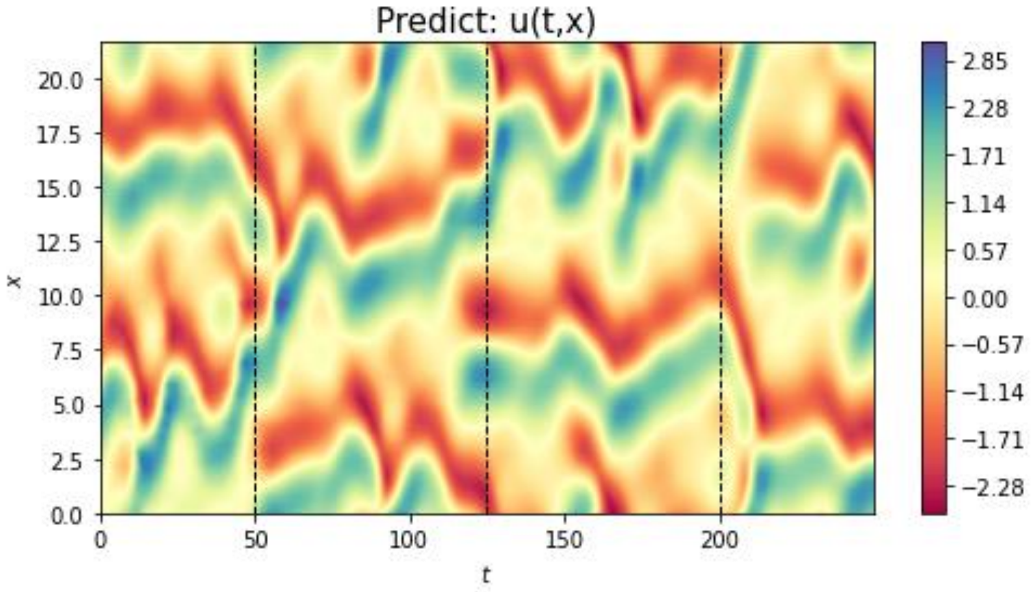
$$u_t + \lambda_1 \cdot uu_x - \lambda_2 \cdot u_{xx} = 0$$




- Target: $u_t + uu_x - (0.01\,/\pi)u_{xx} = 0$
- Prediction: $u_t + 0.97 \cdot uu_x - (0.015\,/\pi)u_{xx} = 0$

# → The Kuramoto-Sivashinsky Equation

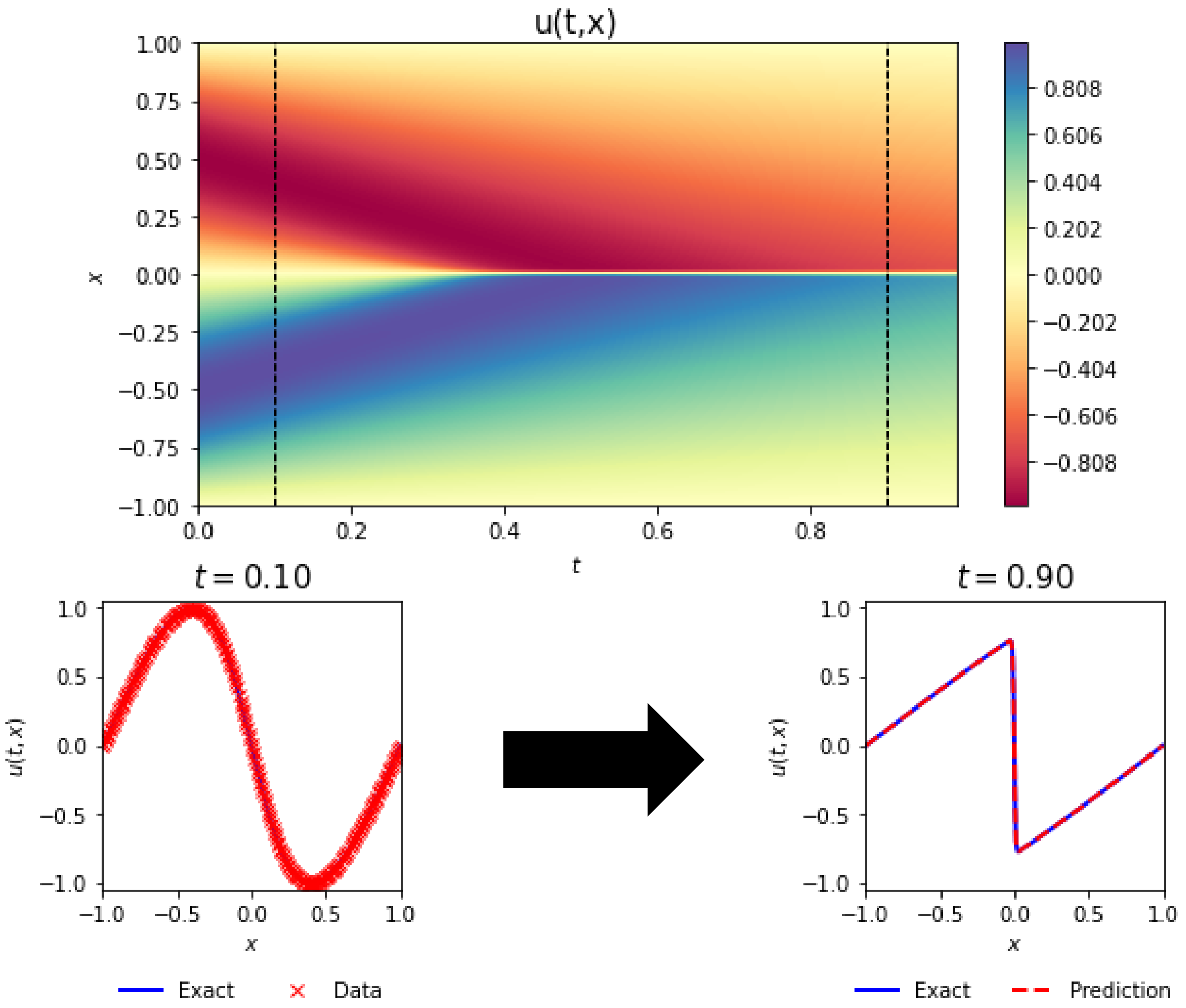
$$u_t + \lambda_1 \cdot uu_x + \lambda_2 \cdot u_{xx} + \lambda_3 \cdot u_{xxxx} = 0$$




- Target: $u_t + uu_x + u_{xx} + u_{xxxx} = 0$
- Prediction: $u_t + 0.093 \cdot uu_x - 0.055 \cdot u_{xx} - 0.001 \cdot u_{xxxx} = 0$

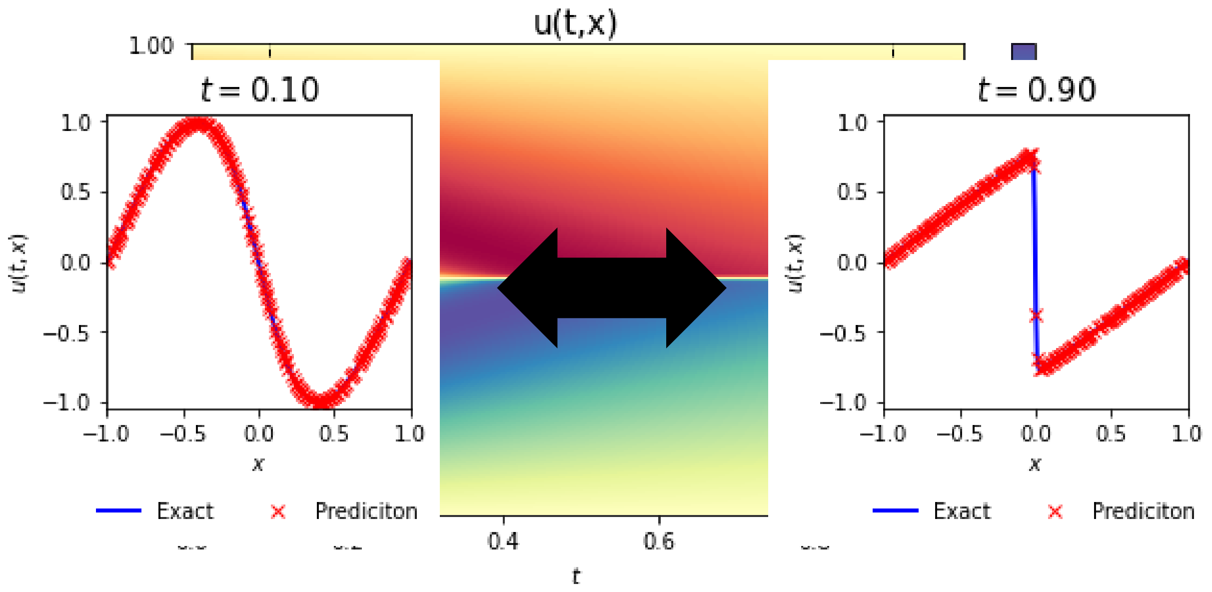
# Discrete-time Inference — The Burgers Equation

$$u_t + uu_x - (0.01/\pi)u_{xx} = 0$$



# Discrete-time Identification — The Burgers Equation

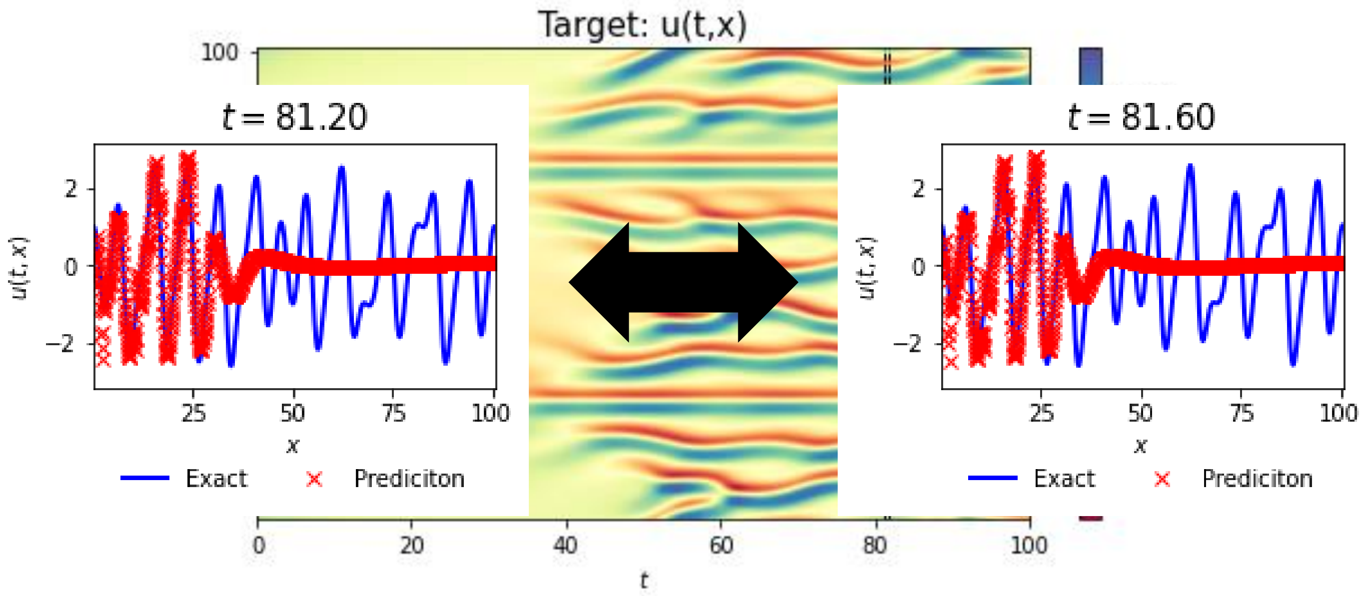
$$u_t + \lambda_1 \cdot uu_x - \lambda_2 \cdot u_{xx} = 0$$




- Target: $u_t + uu_x - (0.01\,/\pi)u_{xx} = 0$
- Prediction: $u_t + 0.998 \cdot uu_x - (0.012\,/\pi)u_{xx} = 0$

# → The Kuramoto-Sivashinsky Equation

$$u_t + \lambda_1 \cdot u u_x + \lambda_2 \cdot u_{xx} + \lambda_3 \cdot u_{xxxx} = 0$$




- Target: $u_t + u u_x + u_{xx} + u_{xxxx} = 0$
- Prediction: $u_t + 0.059 \cdot u u_x + 0.001 \cdot u_{xx} - 2.2 \cdot 10^{-7} \cdot u_{xxxx} = 0$

# Summary and Outlook

- Physics informed neural networks (PINNs) could solve problems with minimal amounts of data

- The frameworks could be applied to different problems involving different partial differential equations (PDEs)

- However, they struggled to solve problems involving higher order PDEs
  - Implementing PINNs in examples where chaotic dynamics are present will require more research

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# Thank you for your interest & attention!


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