About generalized Forward-Backward

Olivier Leblanc

We consider the following minimisation problem:

$$x = \arg\min_u \; F(u) + \sum_i^n G_i(u), \tag{1}$$

where F is a convex differentiable function, and the G_i's are non differentiable but convex so that they have an associated proximal operator. In the *generalized Forward-Backward* algorithm, the iterates go like

$$\begin{aligned} &\text{for } i \in [n] \text{ do} \\ &\quad z_i \leftarrow z_i + \eta \left(\mathrm{prox}_{\frac{\gamma}{\omega_i} G_i}(2x - z_i - \gamma \nabla F(x)) - x \right) \\ &x = \sum_i^n \omega_i z_i, \end{aligned} \tag{2}$$

which when $n = 1$ gives

$$x \leftarrow x + \eta \left(\mathrm{prox}_{\gamma G}(x - \gamma \nabla F(x)) - x \right). \tag{3}$$

One observes (3) is the same as in Algorithm 3.2 in [1]. However, the numerical implementations in PyUnLocBox and PyProximal only handle the case $\eta = 1$. Indeed, their implementation write

$$x \leftarrow \mathrm{prox}_{\gamma G}(x - \gamma \nabla F(x)).$$

Additionally, the generalized FB algorithm does not have any acceleration version to my knowledge.
Above

- γ is the step-size of the gradient descent and proximal steps.
- η is the additional parameter allowing to tune how much the current estimate is updated.
- Finally, one can add an implicit weight parameter in G wrt F, it is generally written λ.

References

[1] Patrick L. Combettes and Jean Christophe Pesquet. Proximal splitting methods in signal processing. *Springer Optimization and Its Applications*, 49:185–212, 2011.