Note for Medlyn stomatal conductance model in FATES

1. Introduction

Previous versions of FATES calculated leaf stomatal resistance is using the Ball-Berry conductance model as described by (Collatz et al., 1991). We provide an alternative way to calculating stomatal conductance, Medlyn stomatal conductance model (Medlyn et al., 2011). The Medlyn model calculates stomatal conductance (i.e., the inverse of resistance) based on net leaf photosynthesis, the vapor pressure deficit, and the CO2 concentration at the leaf surface. Leaf stomatal resistance is calculated as Eq. (1) with the information of symbols listed in Table 1.

 $\frac{1}{r\_{s}}=g\_{s}=g\_{0}×β\_{sw}+1.6(1+\frac{g\_{1}}{\sqrt{D\_{s}}})\frac{A\_{n}}{C\_{s}/P\_{atm}}$ (1)

**Table 1. Information of symbols in Eq. (1)**

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Standard name** | **Unit** |
| $$r\_{s}$$ | Leaf stomatal resistance | s m2 𝜇mol-1 |
| $$g\_{s}$$ | Leaf stomatal conductance | 𝜇mol m-2 s-1 |
| $$g\_{0}$$ | Minimum stomatal conductance | 𝜇mol m-2 s-1 |
| $$β\_{sw}$$ | soil water stress | unitless |
| $$D\_{s}$$ | Vapor pressure deficit at the leaf surface | kPa |
| $$g\_{1}$$ | Slope for the relationship | kPa0.5 |
| $$A\_{n}$$ | Leaf net photosynthesis | 𝜇mol CO2 m-2 s-1 |
| $$C\_{s}$$ | CO2 partial pressure at the leaf surface | Pa |
| $$P\_{atm}$$ | Atmospheric pressure | Pa |

The value for $g\_{0} $= 1000 𝜇mol m-2 s-1 for all PFTs.

$g\_{1}$ is a plant functional type (PFT) dependent parameter. According to the $g\_{1}$ values and PFTs in CLM5, $g\_{1}$ values for different PFTs for FATES model were listed in Table 2.

**Table 2. Stomatal conductance slope parameters in Medlyn model**

|  |  |
| --- | --- |
| **PFT** | $g\_{1}$ **(kPa0.5)** |
| Broadleaf evergreen tropical tree | 4.1 |
| Needleleaf evergreen extratrop tree | 2.3 |
| Needleleaf colddecid extratrop tree | 2.3 |
| Broadleaf evergreen extratrop tree | 4.1 |
| Broadleaf hydrodecid tropical tree | 4.4 |
| Broadleaf colddecid extratrop tree | 4.4 |
| Broadleaf evergreen extratrop shrub | 4.7 |
| Broadleaf hydrodecid extratrop shrub | 4.7 |
| Broadleaf colddecid extratrop shrub | 4.7 |
| Arctic C3 grass | 2.2 |
| Cool C3 grass | 5.3 |
| C4 grass | 1.6 |

1. Numerical implementation

Photosynthesis is calculated assuming there is negligible capacity to store CO2 and water vapor at the leaf surface so that：

 $A\_{n}=\frac{C\_{a}-C\_{i}}{(1.4r\_{b}+1.6r\_{s})P\_{atm}}=\frac{C\_{a}-C\_{s}}{1.4r\_{b}P\_{atm}}=\frac{C\_{s}-C\_{i}}{1.6r\_{s}P\_{atm}}$ (2)

The information of the symbols was listed in Table 3. The terms 1.4 and 1.6 are the ratios of diffusivity of CO2 to H2O for the leaf boundary layer resistance and stomatal resistance. The transpiration fluxes are related as Eq. (3) with the information of symbols listed in Table 3.

 $\frac{e\_{a}-e\_{i}}{r\_{b}+r\_{s}}=\frac{e\_{a}-e\_{s}}{r\_{b}}=\frac{e\_{s}-e\_{i}}{r\_{s}}$ (3)

 $e\_{a}=\frac{P\_{atm}q\_{s}}{0.622}$ (4)

**Table 3. Information of symbols in Eq. (2), (3), and (4)**

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Standard name** | **Unit** |
| $$C\_{a}$$ | Atmospheric CO2 pressure | Pa |
| $$C\_{i}$$ | Internal leaf CO2 partial pressure | Pa |
| $$r\_{b}$$ | Leaf boundary layer resistance | s m2 𝜇mol-1 |
| $$e\_{a}$$ | Vapor pressure of air | Pa |
| $$e\_{i}$$ | Saturation vapor pressure | Pa |
| $$e\_{s}$$ | Vapor pressure at the leaf surface | Pa |
| $$q\_{s}$$ | Specific humidity of canopy air | kg kg-1 |

In the model, an initial guess of $C\_{i}$ is obtained assuming the ratio between$ C\_{i}$ and $C\_{a}$ (0.7 for C3 plants and 0.4 for C4 plants) to calculate $A\_{n}$ based on the Farquhar photosynthesis model (Farquhar et al., 1980). Then Eq. (2) is solved for $C\_{s}$:

 $C\_{s}=C\_{a}-1.4r\_{b}P\_{atm}A\_{n}$ (5)

$e\_{s}$ can be represented from Eq. (3) as:

 $e\_{s}=\frac{e\_{a}r\_{s}+e\_{i}r\_{b}}{r\_{b}+r\_{s}}$ (6)

Where $e\_{i}$ is a function of temperature.

Substitution of $e\_{s}$ into Eq. (1) (according to $D\_{s}=e\_{i}-e\_{s})$ gives an expression for stomatal resistance ($r\_{s}$) as a function of photosynthesis ($A\_{n})$, given here in terms of conductance with $g\_{s}=1/r\_{s}$ and $g\_{b}=1/r\_{b}$

 $g\_{s}^{2}+bg\_{s}+c=0 $ (7)

Where

 $b=-[2\left(g\_{0}×β\_{sw}+d\right)+\frac{\left(g\_{1}d\right)^{2}}{g\_{b}D\_{a}}]$ (8)

 $c=(g\_{0}×β\_{sw})^{2}+\left[2g\_{0}×β\_{sw}+d\left(1-\frac{g\_{1}^{2}}{D\_{a}}\right)\right]d$ (9)

and

 $d=\frac{1.6A\_{n}}{C\_{s}/P\_{atm}}$ (10)

 $D\_{a}=\frac{e\_{i}-e\_{a}}{1000}$ (11)

Stomatal conductance, as solved by Eq. (7), is the larger of the two roots that satisfies the quadratic equation. Values for $C\_{i}$ are given by

 $C\_{i}=C\_{a}-(1.4r\_{b}+1.6r\_{s})P\_{atm}A\_{n}$ (12)

The equations for $C\_{i}$ , $C\_{s}$ , $r\_{s}$ , and $A\_{n}$ are solved iteratively until $C\_{i}$ converges. Iteration will be exited if convergence criteria is met or if at least five iterations are completed.

**References**

Collatz, G.J., Ball, J.T., Grivet, C., Berry, J.A., 1991. Physiological and environmental regulation of stomatal conductance, photosynthesis and transpiration: a model that includes a laminar boundary layer. Agricultural and Forest Meteorology 54: 107-136.

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