We want to prove that P(k + 1) is true whenever P(k) is true, i.e.,

$$P(k+1) : A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Now $$A^{k+1} = A^k . A$$

Since P(k) is true, we have

$$A^{k+1} = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & \cos k\theta\sin\theta + \sin k\theta\cos\theta \\ -\sin k\theta\cos\theta - \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{bmatrix}$$

(by matrix multiplication)

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, P(n) is true for all $n \geq 1$ (by the principle of mathematical induction).

(iii) Proof by cases or by exhaustion

This method of proving a statement $p \Rightarrow q$ is possible only when p can be split into several cases, r, s, t (say) so that $p = r \vee s \vee t$ (where "$\vee$" is the symbol for "OR").

If the conditionals $r \Rightarrow q$;

$s \Rightarrow q$;

and $t \Rightarrow q$

are proved, then $(r \vee s \vee t) \Rightarrow q$, is proved and so $p \Rightarrow q$ is proved.

The method consists of examining every possible case of the hypothesis. It is practically convenient only when the number of possible cases are few.

Example 4 Show that in any triangle ABC,

$$a = b \cos C + c \cos B$$

Solution Let p be the statement "ABC is any triangle" and q be the statement

"$a = b \cos C + c \cos B$"

Let ABC be a triangle. From A draw AD a perpendicular to BC (BC produced if necessary).

As we know that any triangle has to be either acute or obtuse or right angled, we can split p into three statements r, s and t, where