Rank Promotion for GraphBLAS

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FORTRAN terminology

- *Rank* is the number of dimensions of a variable
- Examples (https://gcc.gnu.org/onlinedocs/gcc-4.7.4/gfortran/RANK.html):

```fortran
program test_rank
      integer :: a
      real, allocatable :: b(:,:)

      print *, rank(a), rank(b) ! Prints:  0  2
end program test_rank
```

Rank promotion

- Rank promotion is the automatic conversion of an object to another object of higher rank (e.g., rank 0 to rank 2, rank 1 to rank 2)
- In GraphBLAS
  - Rank promotion preserves the domain
  - A scalar can be promoted to GrB_Vector or GrB_Matrix of the same domain
  - A GrB_Vector can be promoted to GrB_Matrix of the same domain
  - Rank promotion can only be applied to INPUT arguments (probably not masks?)
  - Target rank and shape (after promotion) determined from the other parameters

Tentative list of automatic rank promotions in GraphBLAS

Matrices: $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{M}$
Vectors: $\mathbf{a}, \mathbf{b}, \mathbf{u}, \mathbf{w}, \mathbf{m}$
Scalars: a, b, u, v
$\triangle$ denotes a descriptor
$\mathbb{S}$ is a semiring
$\odot$ is a binary operator used for accumulation
$\oplus$ and $\otimes$ are binary operators, either standalone or from a monoid/semiring
$f(\cdot)$ is a unary operator

Method	Promotions
GrB_mxm($\mathbf{C}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{A}, \mathbf{B}, \triangle$)	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$
	$b \rightarrow \mathbf{B}$
	$\mathbf{b} \rightarrow \mathbf{B}$
GrB_vxm($\mathbf{w}, \mathbf{m}, \odot, \mathbb{S}, \mathbf{u}, \mathbf{A}, \triangle$)	$u \rightarrow \mathbf{u}$
	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$
GrB_mxv($\mathbf{w}, \mathbf{m}, \odot, \mathbb{S}, \mathbf{A}, \mathbf{u}, \triangle$)	$u \rightarrow \mathbf{u}$
	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$
GrB_eWiseMult($\mathbf{w}, \mathbf{m}, \odot, \otimes, \mathbf{u}, \mathbf{v}, \triangle$)	$u \rightarrow \mathbf{u}$
	$v \rightarrow \mathbf{v}$
GrB_eWiseMult($\mathbf{C}, \mathbf{M}, \odot, \otimes, \mathbf{A}, \mathbf{B}, \triangle$)	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$
	$b \rightarrow \mathbf{B}$
	$\mathbf{b} \rightarrow \mathbf{B}$
GrB_eWiseAdd($\mathbf{w}, \mathbf{m}, \odot, \otimes, \mathbf{u}, \mathbf{v}, \triangle$)	$u \rightarrow \mathbf{u}$
	$v \rightarrow \mathbf{v}$
GrB_eWiseAdd($\mathbf{C}, \mathbf{M}, \odot, \otimes, \mathbf{A}, \mathbf{B}, \triangle$)	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$
	$b \rightarrow \mathbf{B}$
	$\mathbf{b} \rightarrow \mathbf{B}$
GrB_assign($\mathbf{C}, \mathbf{M}, \odot, \mathbf{A}, \mathbf{i}, m, \mathbf{j}, n, \triangle$)	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$
GrB_apply($\mathbf{w}, \mathbf{m}, \odot, f(\cdot), \mathbf{u}, \triangle$)	$u \rightarrow \mathbf{u}$
GrB_apply($\mathbf{C}, \mathbf{M}, \odot, f(\cdot), \mathbf{A}, \triangle$)	$a \rightarrow \mathbf{A}$
	$\mathbf{a} \rightarrow \mathbf{A}$

- For purpose of rank promotion, vectors $(\mathbf{a}, \mathbf{b}, \mathbf{u})$ are treated as columns

- The effect of row vectors can be achieved by using the transpose flags in the descriptor

Example: GrB_mxm($\mathbf{C}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{A}, \mathbf{B}, \Delta$)

- Current signature:

```
GrB_Info GrB_mxm(GrB_Matrix              C,
                 const GrB_Matrix  Mask,
                 const GrB_BinaryOp      accum,
                 const GrB_Semiring      op,
                 const GrB_Matrix  A,
                 const GrB_Matrix  B,
                 const GrB_Descriptor    desc);
```

- 8 new signatures (assume transpose flags properly set):
  1. GrB_mxm($\mathbf{C}_{m\times n}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{A}_{m\times k}, \mathbf{b}_k, \Delta$) : $\mathbf{b}_k$ is promoted to matrix $\mathbf{B}_{k\times n}$
  2. GrB_mxm($\mathbf{C}_{m\times n}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{a}_m, \mathbf{B}_{k\times n}, \Delta$) : $\mathbf{a}_m$ is promoted to matrix $\mathbf{A}_{m\times k}$
     GrB_mxm($\mathbf{C}_{m\times n}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{a}_k^T, \mathbf{B}_{k\times n}, \Delta$) : $\mathbf{a}_k$ is promoted to matrix $\mathbf{A}_{m\times k}$
  3. GrB_mxm($\mathbf{C}_{m\times n}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{a}_m, \mathbf{b}_n^T, \Delta$) : $\mathbf{a}_m$ is promoted to matrix $\mathbf{A}_{m\times 1}$, $\mathbf{b}_n$ is promoted to matrix $\mathbf{B}_{1\times n}$
  4. GrB_mxm($\mathbf{C}_{m\times n}, \mathbf{M}, \odot, \mathbb{S}, \mathbf{A}_{m\times k}, b, \Delta$) : b is promoted to matrix $\mathbf{B}_{k\times n}$
  5. …

Explicit rank promotion through GrB_assign

- Current signature (standard matrix variant - $\mathbf{A}_{\text{nrows} \times \text{ncols}}$):

```
GrB_Info GrB_assign(GrB_Matrix          C,
                    const GrB_Matrix     Mask,
                    const GrB_BinaryOp   accum,
                    const GrB_Matrix     A,
                    const GrB_Index      *row_indices,
                    GrB_Index            nrows,
                    const GrB_Index      *col_indices,
                    GrB_Index            ncols,
                    const GrB_Descriptor desc);
```

- Proposed new signature (promote $\mathbf{u}_{\text{nrows}}$ to $\mathbf{U}_{\text{nrows} \times \text{ncols}}$):

```
GrB_Info GrB_assign(GrB_Matrix          C,
                    const GrB_Matrix     Mask,
                    const GrB_BinaryOp   accum,
                    const GrB_Vector     u,
                    const GrB_Index      *row_indices,
                    GrB_Index            nrows,
                    const GrB_Index      *col_index,
                    GrB_Index            ncols,
                    const GrB_Descriptor desc);
```