In[1]:=
```
$LoadPhi = True;
$LoadFeynArts = True;
```

In[3]:=
```
$Configuration = "ChPT2";
$Lagrangians = {"ChPT2"[2], "ChPT2"[4]};
```

In[5]:=
```
<< FeynCalc`;
```

FeynCalc 9.3.1 (stable version). For help, use the documentation center, check out the wiki or visit the forum.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

$Lagrangians : Symbol $Lagrangians appears in multiple contexts {FeynCalc`, Global`}; definitions in context FeynCalc` may shadow or be shadowed by other definitions.

$Configuration : Symbol $Configuration appears in multiple contexts {FeynCalc`, Global`}; definitions in context FeynCalc` may shadow or be shadowed by other definitions.

PHI 1.3 loaded.

Have a look at the supplied examples. If you use PHI in your research, please cite

- F. Orellana, doctoral dissertation, University of Bern, 2003
- F. Orellana, R. Mertig and V. Shtabovenko, in preparation

FeynArts 3.11 (25 Mar 2022) patched for use with FeynCalc, for documentation see the manual or visit www.feynarts.de.

If you use FeynArts in your research, please cite

- T. Hahn, Comput. Phys. Commun., 140, 418–431, 2001, arXiv:hep–ph/0012260

In[7]:=
```
Lagrangian[ChPT2[2]]
```

Out[7]=
$$\frac{1}{4}(f_{\pi\quad})^2 \left(\left\langle \mathcal{D}_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}(\mathcal{U}) * \mathcal{D}_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}(\mathcal{U})^{\dagger}\right\rangle + \left\langle \mathcal{U} * \chi^{\dagger}\right\rangle + \left\langle \chi * \mathcal{U}^{\dagger}\right\rangle\right)$$

In[ ]:=
```
ll = ArgumentsSupply[Lagrangian[ChPT2[2]], x,
    RenormalizationState[0], ExpansionOrder → 4, DropOrder → 4];
```

In[ ]:=
```
lll = DiscardTerms[ll, Retain → {Particle[Pion, RenormalizationState[0]] → 4},
    Method → Expand] // Simplify
```

Out[ ]=

$$\frac{1}{48\,(f_\pi\quad)^2}\Big(-2\left\langle\vec{\pi}\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}\right\rangle+\left\langle\vec{\pi}\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}\right\rangle+3\left\langle\vec{\pi}\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}\right\rangle-\left\langle\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}\right\rangle+\left\langle\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}\right\rangle-2\left\langle\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}\right\rangle+(m_\pi\quad)^2\left\langle\vec{\pi}\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}*\vec{\pi}\cdot\vec{\sigma}\right\rangle\Big)$$

In[ ]:=
```
llle = ExpandU[lll, CommutatorReduce → True] // Simplify
```

Out[ ]=

$$\frac{1}{24\,(f_\pi\quad)^2}\Big(4\left(\vec{\pi}\cdot\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\right)^2-4\,\vec{\pi}\cdot\vec{\pi}\;\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)\cdot\partial_{\text{FeynCalc`PhiStart`Private`ChPT22`Private`}\mu}\left(\vec{\pi}\right)+(m_\pi\quad)^2\left(\vec{\pi}\cdot\vec{\pi}\right)^2\Big)$$

In[ ]:=
```
$IsoIndicesCounter = 0;
```

In[ ]:=
```
llll = llle // IsoIndicesSupply // IndicesCleanup //
    CommutatorReduce[#, FullReduce → True] & // Simplify
```

••• Part : Part 2 of {τ1} does not exist.

••• Part : Part specification 2 [[1]] is longer than depth of object.

••• Part : Part specification 2 [[2]] is longer than depth of object.

••• Sort : Nonatomic expression expected at position 1 in Sort [2].

••• ReplaceAll : {FeynCalc`Channels`Private`CombinationLists ((FeynCalc`Channels`Private`a _?(Equal[≪2≫] ∧ Equal[≪2≫] &))^2 :→ (FeynCalc`Channels`Private`a^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[Cases[≪4≫]] === Sort[{≪1≫}] ∧ Cases[{≪1≫}, ≪1≫, ≪1≫ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

••• Part : Part specification 2 [[1]] is longer than depth of object.

••• General : Further output of Part::partd will be suppressed during this calculation.

••• Sort : Nonatomic expression expected at position 1 in Sort [2].

••• ReplaceAll : {FeynCalc`Channels`Private`CombinationLists ((FeynCalc`Channels`Private`a _?(Equal[≪2≫] ∧ Equal[≪2≫] &))^2 :→ (FeynCalc`Channels`Private`a^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[Cases[≪4≫]] === Sort[{≪2≫}] ∧ Cases[{≪1≫}, ≪1≫, ≪1≫ ∞, Rule[≪2≫]] =!= Sort[{≪2≫}], (FeynCalc`Channels`Private`a _?(≪1≫ &))^2 :→ (≪27≫^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[≪1≫] === ≪1≫ ∧ ≪1≫)} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

Out[ ]=
$$\left\{\frac{\pi^{k_1}\left(\pi^{k_1}\left(\pi^{k_2}\right)^2 (m_\pi^{})^2 + 4\,\partial_{\tau 1}\pi^{k_2}\left(\pi^{k_2}\,\partial_{\tau 1}\pi^{k_1} - \pi^{k_1}\,\partial_{\tau 1}\pi^{k_2}\right)\right)}{24\,(f_\pi^{})^2}\right\} /.$$

```
FeynCalc`Channels`Private`CombinationLists(
  (FeynCalc`Channels`Private`a_?(Count[#1, τ1, ∞, Heads → True] = 1 ∧
        Count[#1, {τ1}[[2]], ∞, Heads → True] = 1 &))^2 :→ (FeynCalc`Channels`Private`a^2 /.
    (Rule @@ #1 &) /@ {Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True], {τ1}}^T) /;
   Sort[Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True]] === Sort[{τ1}] ∧
    Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True] =!= Sort[{τ1}],
  (FeynCalc`Channels`Private`a_?(Count[#1, 2[[1]], ∞, Heads → True] = 1 ∧
        Count[#1, 2[[2]], ∞, Heads → True] = 1 &))^2 :→ (FeynCalc`Channels`Private`a^2 /.
    (Rule @@ #1 &) /@ {Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True], Sort[2]}^T) /;
   Sort[Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True]] === Sort[2] ∧
    Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True] =!= Sort[2]) /.
 FeynCalc`Channels`Private`CombinationLists((FeynCalc`Channels`Private`a_?
    (Count[#1, k1, ∞, Heads → True] = 1 ∧ Count[#1, k2, ∞, Heads → True] = 1 &))^2 :→
  (FeynCalc`Channels`Private`a^2 /. (Rule @@ #1 &) /@
    {Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True], {k1, k2}}^T) /;
   Sort[Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True]] === Sort[{k1, k2}] ∧
    Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True] =!= Sort[{k1, k2}],
  (FeynCalc`Channels`Private`a_?(Count[#1, 2[[1]], ∞, Heads → True] = 1 ∧
        Count[#1, 2[[2]], ∞, Heads → True] = 1 &))^2 :→ (FeynCalc`Channels`Private`a^2 /.
    (Rule @@ #1 &) /@ {Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True], Sort[2]}^T) /;
   Sort[Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True]] === Sort[2] ∧
    Cases[{FeynCalc`Channels`Private`a}, __, ∞, Heads → True] =!= Sort[2])
```

In[ ]:=
```
fields = FieldsSet[QuantumField[Particle[Pion, RenormalizationState[0]]]]
```

Out[ ]=
$$\left\{\pi^{I_1}, \pi^{I_2}, \pi^{I_3}, \pi^{I_4}\right\}$$

In[ ]:=
```
melsimplified = Simplify[FeynRule[llll, fields]]
```

ReplaceAll :

{FeynCalc`Channels`Private`CombinationLists ((FeynCalc`Channels`Private`a _?(Equal[≪2≫] ∧ Equal[≪2≫] &))^2 :> (FeynCalc`Channels`Private`a^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[Cases[≪4≫]] === Sort[{≪1≫}] ∧ Cases[{≪1≫}, ≪1≫, ≪1≫ ‹

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll :

{FeynCalc`Channels`Private`CombinationLists ((FeynCalc`Channels`Private`a _?(Equal[≪2≫] ∧ Equal[≪2≫] &))^2 :> (FeynCalc`Channels`Private`a^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[Cases[≪4≫]] === Sort[{≪2≫}] ∧ Cases[{≪1≫}, ≪1≫, ≪1≫, ∞, Rule[≪2≫]] =!= Sort[{≪2≫}], (FeynCalc`Channels`Private`a _?(≪1≫ &))^2 :> (≪27≫^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[≪1≫] === ≪1≫ ∧ ≪1≫)} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll :

{FeynCalc`Channels`Private`CombinationLists ((FeynCalc`Channels`Private`a _?(Equal[≪2≫] ∧ Equal[≪2≫] &))^2 :> (FeynCalc`Channels`Private`a^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[Cases[≪4≫]] === Sort[{≪2≫}] ∧ Cases[{≪1≫}, ≪1≫, ≪1≫, ∞, Rule[≪2≫]] =!= Sort[{≪2≫}], (FeynCalc`Channels`Private`a _?(≪1≫ &))^2 :> (≪27≫^2 /. (Apply[≪2≫] &) /@ {≪2≫}^T) /; Sort[≪1≫] === ≪1≫ ∧ ≪1≫)} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General : Further output of ReplaceAll::reps will be suppressed during this calculation.

Out[ ]= 0