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Revisiting Oblivious Top-k Selection with Applications to Secure k-NN Classification

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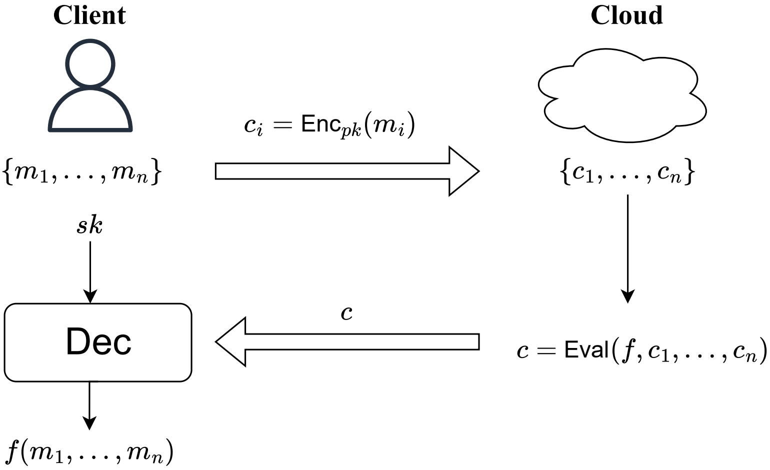
Outline

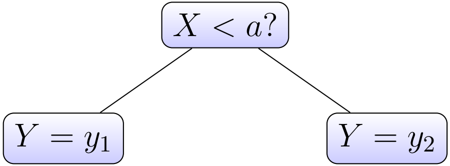
1. Oblivious Algorithms for Secure Computation
2. Oblivious Top-k Selection
3. Application: Secure k-NN Classification
4. Summary and Conclusion

FHE supports secure computation outsourcing



Program expansion in homomorphic branching

- **Program expansion** when converting input-dependent plaintext programs into ciphertext programs
- Example of program expansion:



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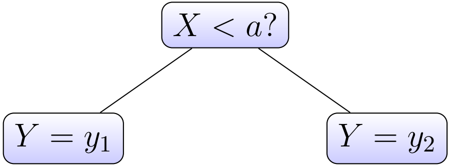

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2. Homomorphically evaluate $Y = (1 - b) \cdot y_1 + b \cdot y_2$

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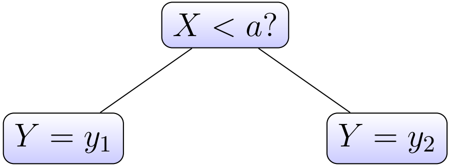

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- *Both* child nodes need to be visited

Oblivious programs and their network realization

Definition

(Data-)oblivious programs are algorithms whose sequence of operations and memory accesses are independent of inputs.

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- Example: sorting d elements
  - Quicksort has complexity $\mathcal{O}(d \log d)$ but is not oblivious
  - Practical oblivious sorting methods have complexity $\mathcal{O}(d \log^2 d)$

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- Example: sorting d elements
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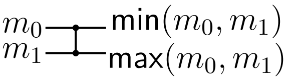
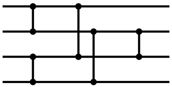
  - Practical oblivious sorting methods have complexity $\mathcal{O}(d \log^2 d)$
- Oblivious programs are visualized as **networks**



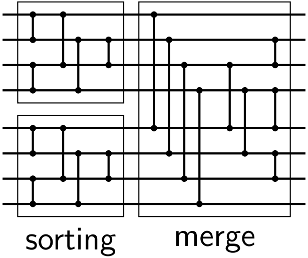
Comparator



Sorting 4 elements obliviously

Example: Batcher's odd-even sorting network

- Built from recursive sortings followed by merge



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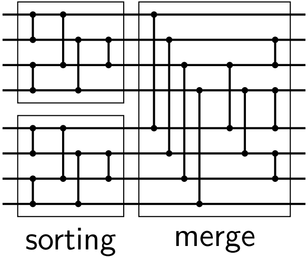

- Batcher's odd-even sorting network has
  - Complexity $S(d) = \mathcal{O}(d \log^2 d)$
  - Depth $\mathcal{O}(\log^2 d)$

Outline

Motivation for Top-k selection problem

Definition

A *Top-k algorithm* selects the k smallest elements from an array of d elements.

- In huge information space, only k most important records are of interest:
  1. Define a proper scoring function
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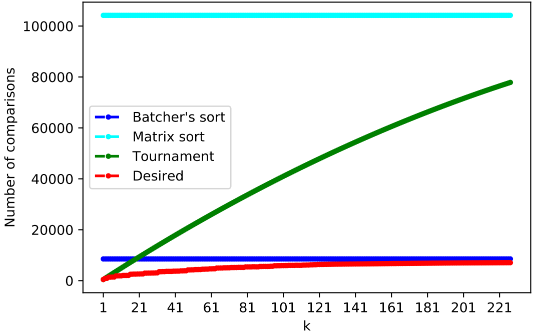
- Example applications include
  - k-nearest neighbors classification
  - Recommender systems
  - Genetic algorithms

Popular oblivious Top-k methods

- First category: oblivious sorting, then discard $d - k$ irrelevant elements
  - Batcher's odd-even merge sort with complexity $\mathcal{O}(d \log^2 d)$ and depth $\mathcal{O}(\log^2 d)$
  - Comparison matrix with complexity $\mathcal{O}(d^2)$ and constant depth



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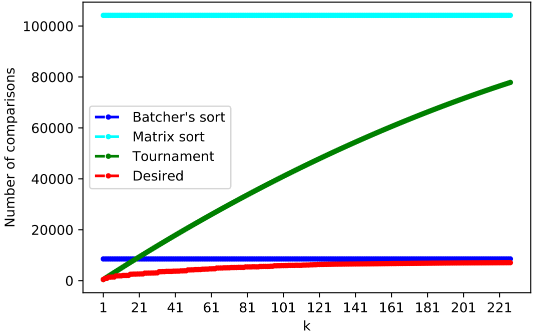
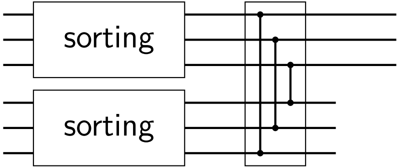
  - Comparison matrix with complexity $\mathcal{O}(d^2)$ and constant depth
- Second category: compute minimum k times
  - Complexity $\mathcal{O}(kd)$ and depth $\mathcal{O}(k \log d)$



Alekseev's oblivious Top-k for $2k$ elements

- Realization using two building blocks:
  - Sorting network of size k
  - Pairwise comparison



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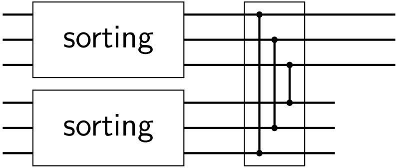
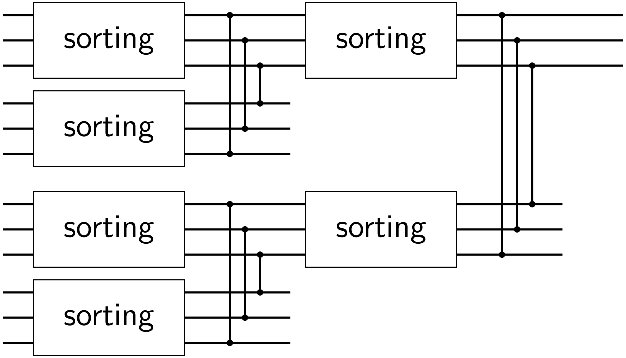


- Can be generalized to Top-k out of d elements

Alekseev's oblivious Top-k for d elements

- Top-k complexity is $\mathcal{O}(d \log^2 k)$ if $S(k) = \mathcal{O}(k \log^2 k)$



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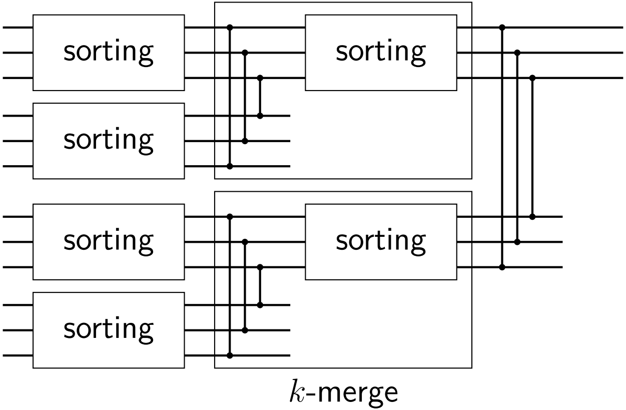

- Realizes k-merge as pairwise comparison + sorting: complexity $k + S(k)$

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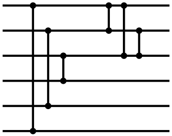
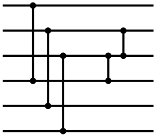
Improvement I: order-preserving merge

- Batcher's odd-even sorting network uses alternative merge
  - Truncate to k-merge by removing redundant comparators
  - Complexity reduction from $\mathcal{O}(k \log^2 k)$ to $\mathcal{O}(k \log k)$

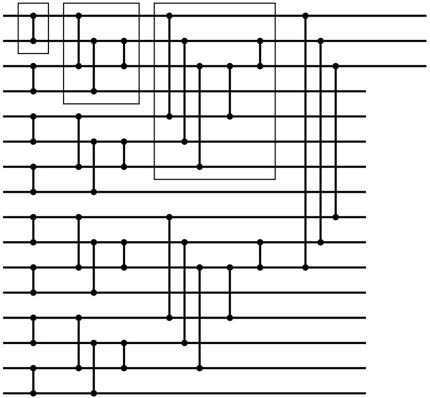


(a) Alekseev's 3-merge



(b) Our 3-merge

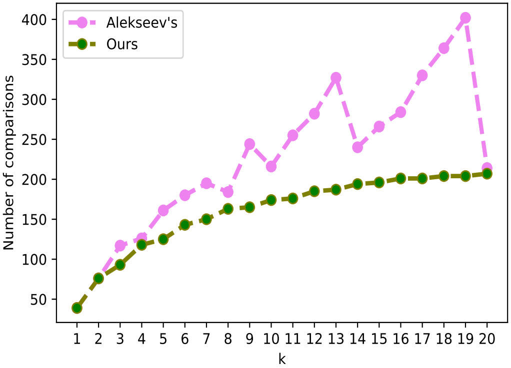
Improvement I: oblivious Top-k from truncation



Network realization for Top-3 of 16 elements

Improvement I: comparison

- Same asymptotic complexity as Alekseev: $\mathcal{O}(d \log^2 k)$ comparators
- Our solution contains fewer comparators in practice



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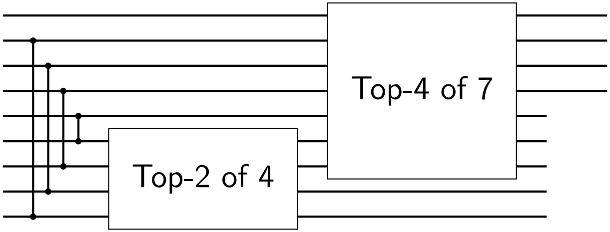
Revisiting Yao's oblivious Top-k

- Andrew Yao improved Alekseev's Top-k using an unbalanced recursion



Selecting Top-4 of 9 elements using Yao's method

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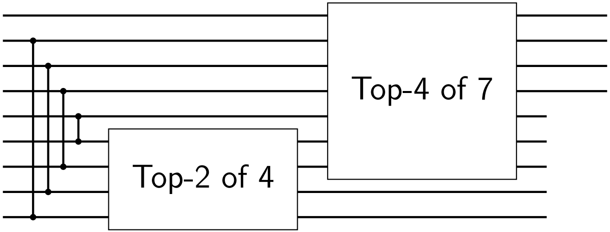
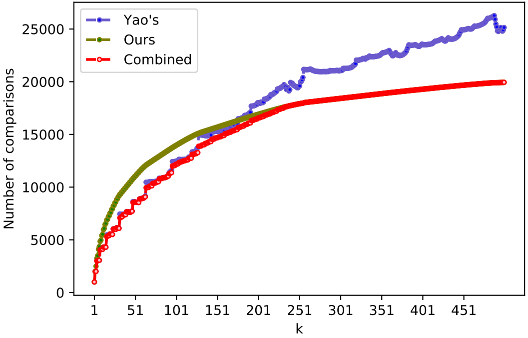


Selecting Top-4 of 9 elements using Yao's method

- $k \ll \sqrt{d}$: complexity is $\mathcal{O}(d \log k)$
- $k \gg \sqrt{d}$: complexity is asymptotically higher than $\mathcal{O}(d \log^2 k)$

Improvement II: combining our method with Yao's

- Combined network recursively calls our or Yao's method
- Slightly improves on the better method in some cases



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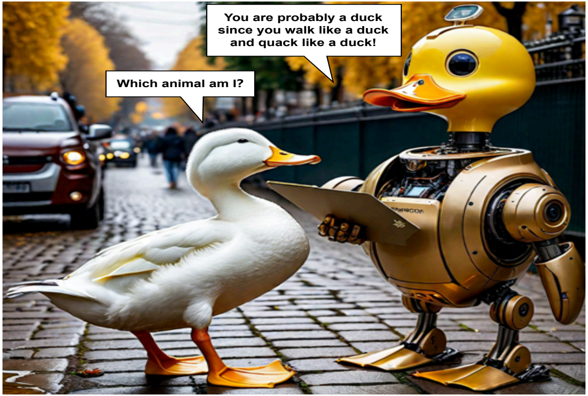
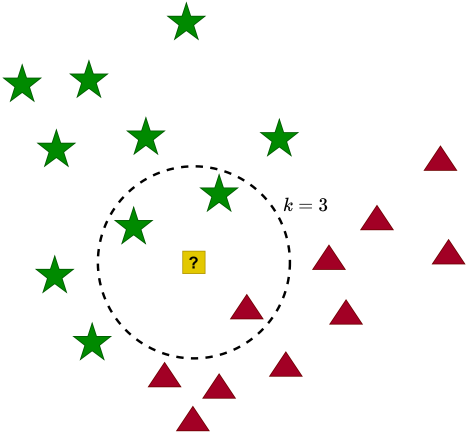
Introduction to k-Nearest Neighbors

- Simple machine learning algorithm with broad applications
  - Plagiarism detection, image classification, intrusion detection, ...
  - Lazy learning: no training phase



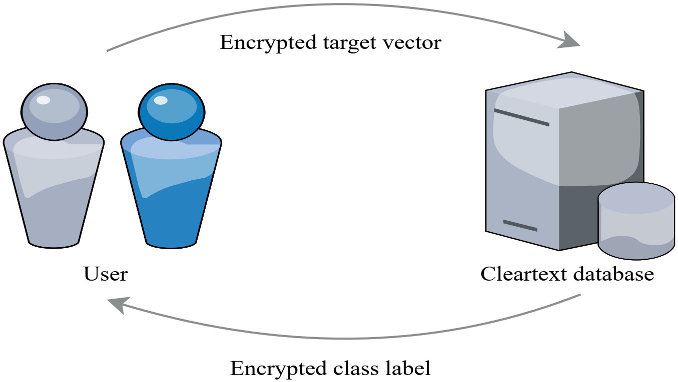
Introduction to k-Nearest Neighbors

- Three-step method:
  1. Compute distance between target vector and d database vectors
  2. Find k closest database vectors and corresponding labels
  3. Class assignment is majority vote of these k labels



Secure k-NN threat model

- Client sends encrypted k-NN query to server
- Server returns encrypted classification result



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   - Each comparator uses two programmable bootstrappings

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$$\text{using } i = \arg\min(\text{dist}_0, \text{dist}_1)$$

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Performance for MNIST dataset

- Implementation in TFHE-rs

k	d	Comparators [ZS21]†	Comparators Ours	Duration (s) [ZS21]†	Duration (s) Ours	Duration (s) Speedup
3	40	780	93	30	**18**	**1.7**×
	457	104196	1136	4248	**202**	**21.0**×
	1000	499500	2493	20837	**441**	**47.2**×
$\lfloor\sqrt{d}\rfloor$	40	780	143	33	**28**	**1.2**×
	457	104196	3412	4402	**530**	**8.3**×
	1000	499500	9121	21410	**1252**	**17.1**×

†Zuber and Sirdey: Efficient homomorphic evaluation of k-NN classifiers

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  - $\mathcal{O}(d \log^2 k)$ in general
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- An oblivious Top-k algorithm with complexity
  - $\mathcal{O}(d \log^2 k)$ in general
  - $\mathcal{O}(d \log k)$ for small $k \ll \sqrt{d}$
- Implementation of a secure k-NN classifier in TFHE-rs
  - Feasible for database of 1000 records: 47$\times$ faster than [ZS21]
- Top-k is an important submodule for various other applications

Thank you for your attention!

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