The Variable Infiltration Capacity (VIC) surface runoff parameterization is based on sub-grid variability in the maximum storage capacity (infiltration capacity) of soils. The sub-grid variability in infiltration capacity is defined using a Pareto distribution (Moore, 2007)

$$F(c) = 1 - \left(1 - \frac{c}{c_{\max}}\right)^{b} \tag{1}$$

$$f(c) = \frac{dF(c)}{dc} = \frac{b}{c_{\max}}\left(1 - \frac{c}{c_{\max}}\right)^{b-1} \tag{2}$$

where c is the infiltration capacity at a given point in space, $c_{\max}$ is the maximum infiltration capacity, and b is the shape of the Pareto distribution.

The basin average storage can be obtained by integrating over the sub-grid probability distribution of soil stores (Moore, 2007), i.e.,

$$S(c^*) = \int_0^{c^*} \left(1 - F(c)\right) dc \tag{3}$$

which has the analytical solution

$$S(c^*) = S_{\max}\left\{1 - \left(1 - \frac{c^*}{c_{\max}}\right)^{b+1}\right\} \tag{4}$$

where $S_{\max} = c_{\max} \big/ (b+1)$.

We are interested in the calculating infiltration into the soil given a flux at the upper boundary, q_{top}, over a finite time interval Δt. Following equation (3), the change in storage over the time interval is

$$S^{n+1} - S^{n} = \int_{c^*(t)}^{c^*(t)+q_{top}\Delta t} \left(1 - F(c)\right) dc \tag{5}$$

where the superscripts n and $n+1$ define the start and end of the time step respectively.

The partitioning between surface runoff and infiltration is typically solved using equations (4) and (5). Given the volume flux at the upper boundary, $q_{top}\Delta t$, the storage at the end of the time step is

$$S^{n+1} = S_{\max}\left\{1-\left(1-\frac{c^*(t)+q_{top}\Delta t}{c_{\max}}\right)^{b+1}\right\} \tag{6}$$

and the infiltration and surface runoff is

$$q_{in} = \frac{S^{n+1}-S^n}{\Delta t} \tag{7}$$

$$q_{satx} = q_{top} - \frac{S^{n+1}-S^n}{\Delta t} \tag{8}$$

where S^n is the storage at the start of the time step.

The VIC implementation is consistent with the formulation in Moore (2007). Expanding equation (6)

$$S^{n+1} = S_{\max} - S_{\max}\left(1-\frac{c^*+q_{top}\Delta t}{c_{\max}}\right)^{b+1} \tag{9}$$

and substituting S^{n+1} in equation (8), then

$$q_{satx}\Delta t = q_{top}\Delta t - S_{\max} + S^n + S_{\max}\left(1-\frac{c^*+q_{top}\Delta t}{c_{\max}}\right)^{b+1} \tag{10}$$

Note that in VIC the fractional saturated area is given by

$$F(S) = 1-\left(1-\frac{S}{S_{\max}}\right)^{\frac{b}{b+1}} \tag{11}$$

obtained through use of equation (1) after solving equation (4) for c^*.