

To simplify things I set  $ktens = 0$ .

I think that we only need to look at one of the stress equation (the replacement pressure affects only one term in the equation for  $\sigma_1$ ). Hence, I start with

$$\frac{\partial \sigma_1}{\partial t} + \frac{\sigma_1}{2T} + \frac{P_R}{2T} = \frac{P}{2T\Delta} D_D, \quad (1)$$

where  $D_D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ ,  $\Delta$  is also a deformation,  $T$  is the EVP damping time scale,  $P$  is the ice strength and  $P_R$  is (I would argue) the replacement pressure as we will see later...

With the time discretization of the sigmas we have

$$\frac{\sigma_1^{k+1} - \sigma_1^k}{\Delta t_e} + \frac{\sigma_1^{k+1}}{2T} + \frac{P_R}{2T} = \frac{P}{2T\Delta} D_D, \quad (2)$$

rearranging the terms...

$$\sigma_1^{k+1} \left[ \frac{1}{\Delta t_e} + \frac{1}{2T} \right] = \frac{\sigma_1^k}{\Delta t_e} - \frac{P_R}{2T} + \frac{P}{2T\Delta} D_D, \quad (3)$$

I introduce  $\alpha = \frac{\Delta t_e}{2T}$  and write the last equation as

$$\frac{\sigma_1^{k+1}}{denom1} = \sigma_1^k - P_R \alpha + \frac{P\alpha}{\Delta} D_D, \quad (4)$$

with  $denom1 = (1 + \alpha)^{-1}$ .

I will use the notation of the code. Let's just look at the north-east corner.  $\sigma_1$  is  $stressp_1$ . Hence we have

$$stressp_1 = \left( stressp_1 - P_R \alpha + \frac{P\alpha}{\Delta} D_D \right) denom1, \quad (5)$$

$P$  in the code is strength and  $\frac{P\alpha}{\Delta}$  becomes  $\alpha * c0ne$ . Note that  $c0ne$  now includes the capping of the viscous coefficient ( $P/\Delta = 2\zeta$ ). The capping however is not the replacement pressure...at this point we have the same rheology as in Hibler 1979 (except of course the elastic term).

With  $c1ne = \alpha c0ne$  we have

$$stressp_1 = (stressp_1 - P_R \alpha + c1ne * D_D) denom1, \quad (6)$$

In the code  $D_D = divune$  and I rearrange the terms so that we have

$$stressp_1 = (stressp_1 + c1ne * divune - P_R \alpha) denom1, \quad (7)$$

You can see that the equation is the same as in the code except for the last term on the RHS because we have not implemented the replacement pressure. If we did not have the replacement pressure we would have

$$stressp_1 = (stressp_1 + c1ne * divune - P\alpha) denom1 = (stressp_1 + c1ne * divune - \alpha * strength) denom1, \quad (8)$$

The idea of the replacement pressure is to make sure the ice is not put in motion when it is at rest and there is no forcing. With equation 8, the first two terms on the RHS would be zero but not the last one (i.e.,  $-\alpha * strength$ ). This is why we need the replacement pressure. The idea is to replace  $strength$  by  $c0ne * Deltane$ . In this case, if  $Deltane$  is small  $c0ne * Deltane$  tends toward zero (this is what we want so that the ice would not be set in motion) while if the deformations are large (plastic) then we have  $c0ne * Deltane \sim strength$ .

So if we have  $P_R = c0ne * Deltane$  in equation 7 we find

$$stressp_1 = (stressp_1 + c1ne * divune - \alpha c0ne * Deltane) denom1, \quad (9)$$

which finally becomes

$$stressp_1 = (stressp_1 + c1ne [divune - Deltane]) denom1, \quad (10)$$

This is exactly what we have in the code.